VERMONT STATE MATHEMATICS COALITION TALENT SEARCH October 4, 2005

Test 1 of the 2005 - 2006 school year (Test 2 arrives at schools November 15, 2005)

Student Name _____ School _____

Grade _____ Math Department Head _____

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by November 1, 2005 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: http://www.state.vt.us/educ/vsmc)

1. There are 7 student lifeguards A, B, C, D, E, F, G who each work one afternoon per week at the town pool, none of them working on the same day. A's day at the pool is the day after C's. B's day is three days before G's. D's work day is three days after the day before E's. F's day is mid-way between B's and C's and is on a Thursday. List the work days of the students from Sunday through Saturday.
Answer:

Sun. Mon. Tues. Wed. Thurs. Fri. Sat.

____ ____ ____ ____ ____ ____ ____

2. The leftmost digit of a whole number is 2. When this 2 is moved to become the rightmost digit (and the order of the other digits is not changed), then the new number is three times the original number. Find the smallest whole number for which this is true.

Answer: _____

3. Forty consecutive odd numbers are listed from smallest S to largest L. The sum of these forty numbers is a perfect cube. Find the smallest possible value for S.

Answer: _____

4. In the 5 by 5 array, the numbers in every row, every column and both diagonals form arithmetic progressions. You are given that N(1, 3) = 33, N(3, 1) = 22, and N(5, 4) = 16. Find the sum of the 25 numbers in the array.

		33		
22				
			16	

Answer: _____

5. Two triangles lying in perpendicular planes have a 27 inch side in common. The sides of one triangle are 21 in., 24 in., 27 in. The sides of the second triangle are 27 in., 30 in., and x in., where $x > 30$. When the distances between the vertices of the triangles are measured, the largest distance is 34 in. Compute x.

Answer: _____

6. A trapezoid will be constructed to have its four sides of lengths 3, 4, 6 and 9 inches. It is possible to construct three non-congruent trapezoids from sides of these lengths.
a) Select the trapezoid that has the largest area of the three.

b) Using that trapezoid of part a, draw a line segment parallel to the two parallel sides so that the trapezoid is divided into two sections that have the same perimeter. Find the exact area of the smaller of those two sections.

Answer: a)_____, b)_____

7. Given that $1^5 + 2^5 + 3^5 + \cdots + n^5 = \frac{1}{6}n^6 + an^5 + bn^4 + cn^2$, evaluate $2005(a + b - c)$.

Answer: _____

8. If the second term of an arithmetic progression is decreased by 2 and the fourth term is increased by 10, then the first four terms form a geometric progression. Which term of the arithmetic progression is 2008?

Answer: _____