

Test 3 of the 2005 - 2006 school year

(Test 4 arrives at schools February 14, 2006)

Student Name \_\_\_\_\_

School \_\_\_\_\_

Grade \_\_\_\_\_

Math Department Head \_\_\_\_\_

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by January 31, 2006 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: <http://www.state.vt.us/educ/vsmc>)

1. To make their tennis competition more exciting, Andre and Venus bet \$1.00 on each game they play. When they finished playing, Andre had won a total of 3 games, and Venus had won a total of \$3.00. How many games did they play?

Answer: \_\_\_\_\_

2. The students taking senior math decided to send birthday cards to their teacher. The cards will require stamps costing 37¢. The stamps that might be placed on an envelope have the denominations 1¢, 2¢, 3¢, 4¢, 5¢, 10¢, 15¢, 23¢, and 37¢. The students decided to look for different combinations of stamps for each envelope. Kim's suggestion was using 6 or fewer stamps on each envelope and using at most 3 of a particular denomination on an envelope. When they counted how many different ways envelopes could be stamped this way, they discovered, amazingly enough, that the number of ways was the same as the number of students taking senior math. How many different ways could stamps be put on the envelopes according to Kim's suggestion?

Answer: \_\_\_\_\_

3. Find the smallest value of  $k$  for which  $\frac{(k!) \cdot (k!)}{(k+6)!}$  is an integer.

Answer: \_\_\_\_\_

4. a) The lengths of the sides of a triangle are 6, 10, and 12. Find the sum of the squares of the lengths of the medians of the triangle.

b) The lengths of the sides of a triangle are  $2a$ ,  $2b$ , and  $2c$ . Find the sum of the squares of the lengths of the medians of the triangle.

Answer: a) \_\_\_\_\_ b) \_\_\_\_\_

5. A circle was inscribed within the triangle ABC. The circle is tangent to AB at point P with AP = 5 and BP = 3. The area of triangle ABC is  $30 \cdot \sqrt{3}$ .

a) Find the perimeter of triangle ABC.

b) Find the measure of angle A.

Answer: a) \_\_\_\_\_

b) \_\_\_\_\_

6. In the given nine by nine Latin Square, each of the digits 1 through 9 is used in every row and in every column. The nine by nine Latin Square is composed of nine blocks with each having 3 rows and 3 columns. Each of these arrays contains the digits 1 through 9. (This description makes the Latin square a sudoku puzzle.) This Latin square is special because of this feature of all nine of the 3 by 3 blocks: the sum of the three numbers is 15 in every row and every column of any 3 by 3 array.

	<b>8</b>							
			<b>3</b>	<b>8</b>			<b>2</b>	
	<b>6</b>							
								<b>2</b>
	<b>7</b>		<b>9</b>			<b>8</b>		
<b>4</b>								
<b>8</b>								
<b>6</b>			<b>8</b>		<b>3</b>			

7. The three roots of the equation  $x^3 - 35x^2 + 55x - 80 = 0$  are  $a$ ,  $b$ , and  $c$ .

Evaluate  $(a + b) \cdot (a + c) \cdot (b + c)$ .

Answer: \_\_\_\_\_

8. In the system of simultaneous equations

$$abcd + abce + abde + acde = -264$$

$$abcd + abce + acde + bcde = -24$$

$$abcd + abce + abde + bcde = 24$$

$$abcd + abde + acde + bcde = 248$$

$$abce + abde + acde + bcde = -16$$

the numbers  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  are rational. Evaluate  $70a - 50b + 60c - 20d + 80e$ .

Answer: \_\_\_\_\_