

Test 4 of the 2005 - 2006 school year (Test 4 completes the Talent Search Contest for '05-'06)

Student Name _____ School _____

Grade _____ Math Department Head _____

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by March 14, 2006 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: <http://www.state.vt.us/educ/vsmc>)

1. George Washington was born 11 years before Thomas Jefferson. In 1770, Washington's age was 3 more than 7 times the age of Jefferson in 1748. How old was Jefferson in 1776?

Answer: _____

2. For positive integers x , $P(x)$ denotes the smallest prime number greater than x , and $s(x)$ denotes the greatest prime number less than x . For example, $P(13) = 17$, and $s(13) = 11$.

Evaluate

$s(s(3000)) - P(s(2000)) + P(P(3100)) - s(P(2088))$.

Answer: _____

3. ABCD is a trapezoid with $DA \perp CD$. $AD = 24$, $AB = 32$, and $CD = 64$. A fold is made in the trapezoid so that points C and B coincide. Find the length of the fold.

Answer: _____

4. The triangle ABC, then $AC = BC$. Point D lies on side AC so that $AB = CD$. If angle C measures 20° , then find the measure of angle ADB.

Answer: _____

5. On a standard 8 by 8 chessboard, it is possible to place 8 queens so that no queen can capture another queen. (This means that no two queens are in the same row or the same column or along the same diagonal.) In fact, there are 92 different ways to do this. You are shown a 4 by 4 chess board with 4 queens (written with notation A) placed so that no two queens can attack each other. On another 4 by 4 chessboard, four queens (written with notation B) have been placed so that no two of them can attack each other.

	A		
			A
A			
		A	

positions of
four queens A

		B	
B			
			B
	B		

positions of
four queens B

	A	B	
B			A
A			B
	B	A	

positions of
queens
A and B

answer

You are asked to use 5 by 5 chess boards. Place five queens in positions marked with 5 As, 5 queens in positions marked with 5 Bs, 5 queens in positions marked with 5 Cs, 5 queens in positions marked with 5 Ds, and 5 queens in positions marked with 5 Es. Place the queens so that no two of them can attack each other. Place the queens so that they can be transferred to one 5 by 5 chessboard so that the 25 queens fill the 25 squares. Your answer, *included above*, should show one 5 by 5 chessboard with the 25 squares marked with 5 of each letter A, B, C, D, and E. (This can be done in more than one way—you should include just one solution.)

6. The function $f_1(x) = \frac{2x-1}{x+1}$, and for $n = 1, 2, 3, 4, \dots$, then

$$f_{(n+1)}(x) = f_1(f_n(x)). \quad \text{Evaluate } f_{2006}(x).$$

Answer: _____

7. The interior angles of a polygon have measures $170^\circ, 160^\circ, 150^\circ, 140^\circ, \dots$ down to some smallest angle. The numbers 170, 160, 150 of the list form an arithmetic progression. Find the number of sides of the polygon.

Answer: _____

8. $S(n) = \sum_{k=1}^n (2k-1) \cdot i^k$, where $i^2 = -1$. Write $S(45)$ in the form $a + bi$.

Answer: _____