VERMONT STATE MATHEMATICS COALITION TALENT SEARCH November 21, 2006

Test 2 of the 2006 - 2007 school year (Test 3 arrives at schools January 9, 2007)

Student Name _____ School _____

Grade _____ Math Department Head _____

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by December 19, 2006 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. For Coalition information and a copy of the test:
http://www.vermontinstitutes.org/vsmc/talent-search/

1.

Arrange the numbers 1 through 15, placing them in the boxes so that the two numbers in any adjacent boxes have a sum that is a square number. Do this in a way that the number in the box at the left is less than the number in the box at the right.

2. Beginning with 1, all the positive integers are written successively as 12345678910111213… .

What digit appears in the 2007^{th} position?

Answer: _____

3. An eight digit number contains two 1s, two 2s, two 3s and two 4s. The 1s are separated by one digit; the 2s are separated by two digits; the 3s are separated by 3 digits; and the 4s are separated by 4 digits.

Find the sum of all possible eight digit numbers with this property.

Answer: _____

4. A sequence of integers is defined as follows $a_1 = 1$, $a_2 = 1$,
and $a_n = \frac{1}{4}a_{n-2} + \frac{1}{3}a_{n-1}$ for $n = 3, 4, 5, 6, \ldots$.

Let $S = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + \ldots$. Find S.

Answer: _____

5. If A, B, and C are digits, the following four numbers form an arithmetic progression: AB4, B03, B3C, BA1.

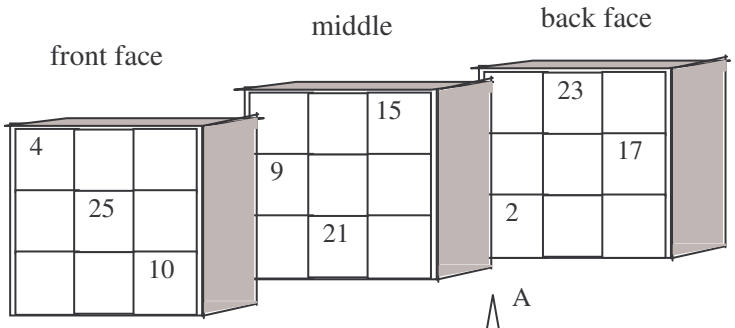
Find the numerical value of the 47th term of the arithmetic progression.

Answer: _____

6. A large cube is divided into 27 smaller cubes, with each of the numbers 1 through 27 used to number the smaller cubes. The cubes are placed together in three groups of nine cubes. In all this creates nine rows and nine columns in which every row and every column has the same sum. Some of the numbers are shown.

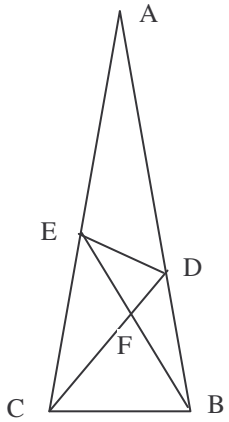
Fill in the other numbers so that you can find the sum of the numbers on the eight corners of the large cube.

Answer: _____



7. In an isosceles triangle ABC, AB = AC and ∠ A measures 20 degrees. Points E and D are placed along the sides AC and AB respectively so that AE = BE and ∠ ACD = 50º.
Find the measure of ∠ BED.

Answer: _____



8. Graphed for you are $y = x^2 + mx + n$, $y = ax$, and $y = bx$, where $b > a$. The line $y = ax$ intersects the parabola at two points, and the line $y = bx$ intersects the parabola at two points. Each of these four points is projected down to a point (with the same x-coordinate) on the x-axis. In pairs, these four points on the x-axis are used (as pictured) to create two intervals on the x-axis with lengths d_1 and d_2.

Find $d_1 - d_2$.

Answer: _____