VERMONT STATE MATHEMATICS COALITION TALENT SEARCH February 20, 2007

Test 4 of the 2006 - 2007 school year (This Test 4 concludes testing for the 2006-2007 year.)

Student Name ________________________ School ____________________________

Grade ________ Math Department Head _______________________________

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by March 20, 2007 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. For Coalition information and a copy of the test:
http://www.vtmathcoalition.org /

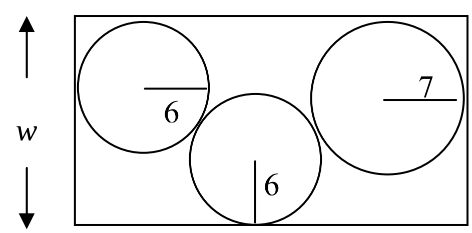
1. The number $\frac{62}{27}$ can be written in the form $v + \cfrac{1}{w + \cfrac{1}{x + \cfrac{1}{y + \cfrac{1}{z}}}}$

where v, w, x, y, z are positive integers. Evaluate $242 \cdot v + 164 \cdot w + 213 \cdot x + 191 \cdot y + 207 \cdot z$.

Answer: ______________

2. Three tangent circles are inscribed in a rectangle. The three circles have radii 6, 6, and 7 (as shown) and the rectangle has width $w = 18$. Find the length of the rectangle.

Answer: ______________



3. The number 2 and the Golden Ratio $\Phi = \frac{1+\sqrt{5}}{2}$ are two roots of the equation

$x^5 - 4 \cdot x^4 + a \cdot x^3 + b \cdot x^2 + c \cdot x - 24 = 0$, where a, b, and c are integers.

Find the sum $a + b + c$.

Answer: ______________

4. The function f has the properties that f(0) = 1, and for any positive integer n, then $f(n) = n \cdot f(n-1)$.

Find all integral values of n, where $n>4$, for which $\frac{5f(n)}{f(n-3)} = \frac{2f(n-1)}{f(n-5)}$.

Answer: _______________

5. For $w \geq 0$ and for x, y, and z, find the smallest value of x satisfying the following system of equations.

$y = x - 2007$

$z = 2y - 2007$

$w = 3z - 2007$

Answer: _______________

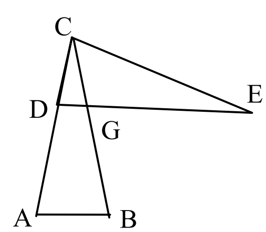
6. Suppose that $P(x)$ is a polynomial of degree 2006 with $P(n) = \frac{1}{n}$ for $n = 1, 2, 3, 4, \ldots, 2007$. Evaluate $P(2008)$.

Answer: _______________

7. An isosceles triangle has sides of lengths $\sin(\theta)$, $\sqrt{\sin(\theta)}$, and $\sqrt{\sin(\theta)}$. The angle opposite the side of length $\sin(\theta)$ has measure θ. Find the numerical value of the area of the triangle.

Answer: _______________

8. You are given that Δ ABC is congruent to Δ CDE, and that $\angle A = \angle B = 80^\circ$. Suppose that AC = 1 and $DG = \frac{2\sin(10^\circ)}{P + Q\sin^2(10^\circ)}$.



Evaluate $P + Q$.

Answer: _______________