VERMONT STATE MATHEMATICS COALITION TALENT SEARCH October 2, 2007

Test 1 of the 2007 - 2008 school year (*Test 2 will arrive at schools on November 13, 2007*)

PRINT NAME: ____________________________ Sign Name: _____________________________

School _________________________________Grade _______

Math Department Head _______________________________

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by October 30, 2007 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. For Coalition information and a copy of the test:
http://www.vtmathcoalition.org /

1. A “math competition number” is a positive integer that has two more prime factors than its predecessor. For example,
(i) 8 is a math competition number since $8 = 2 \cdot 2 \cdot 2$ (three prime factors) and 7 = 7 (one prime factor), but
(ii) 24 is not a math competition number since $24 = 2 \cdot 2 \cdot 2 \cdot 3$ (four prime factors), and 23 = 23 (one prime factor).

Of the first twelve math competition numbers, how many of them are divisible by 8?

Answer: _______________

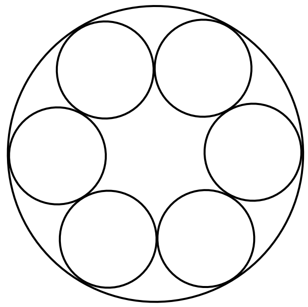
2. There are four integers between 100 and 1000 that are equal to the sum of the cubes of their digits. Three of them are 153, 371, and 407. Find the fourth such integer.

Answer: _______________

3. An integer n has a square whose tens digit is odd. Find the units digit of n^2.

Answer: _______________

4. Six congruent circles are arranged inside a larger circle so that each small circle is tangent to two other small circles and is tangent to the large circle. The radius of the large circle is 2007 cm. Find the radius of the small circles.



Answer: _______________

5. For the equation $x^5 - 12x^4 + ax^3 + bx^2 + cx - 64 = 0$,
all of its roots are positive real numbers. Find the sum $a + b + c$.

Answer: _______________

6. There are four integers a, b, c, and d, with $a > b > c > d$. When the integers are added three at a time, the resulting sums are 20, 22, 24, and 27.
Evaluate $61\cdot(a + c) + 70\cdot(b + d)$.

Answer: _______________

7. Find all positive integers n (with $n > 1$) for which $n!$ is divisible by $\sum_{i=1}^{n} i$.

Answer: ______________________________

8. The rational number x satisfies the equation $\left(\frac{4}{\sqrt{3}-\sqrt{2}}\right)^{4-x} = \left(80+32\sqrt{6}\right)^{x}$.

Find x.

Answer: _______________