VERMONT STATE MATHEMATICS COALITION TALENT SEARCH November 13, 2007

Test 2 of the 2007 - 2008 school year (*Test 3 will arrive at schools on January 7, 2008*)

PRINT NAME: ___________________________ Sign Name: ____________________________

School ________________________________Grade _______

Math Department Head ______________________________

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by December 11, 2007 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. For Coalition information and a copy of the test:
http://www.vtmathcoalition.org /

1. There are n consecutive integers starting with n itself as the smallest of the integers. The sum of the n integers is 92. Find n.

Answer: ______________________

2. Triangle ABC has a right angle at C. P is an interior point of the triangle for which $\angle APC = \angle BPC = \angle APB$. If AP = 10, CP = 6, and BP = d, find d.

Answer: ______________________

3. In a quadrilateral ABCD, the points M, N, P, and Q are the midpoints of the sides AB, BC, CD, and DA respectively. MN separates the quadrilateral into two sections whose areas are in the ratio 1 : 6. The line segment QP separates ABCD into two sections whose areas are in the ratio $u : v$. Find the ratio $u : v$.

Answer: ______________________

4. Find all real numbers *x* for which $\left(x+2\sqrt{13}\right)^{\frac{1}{3}} + \left(x-2\sqrt{13}\right)^{\frac{1}{3}} = 1$.

Answer: ______________________

5. In parallelogram ABCD, AB > BC. Point P lies on diagonal AC so that P is equidistant from both AB and BC. Similarly, point Q lies on AC so that Q is equidistant from both CD and AD. If $PQ = \frac{AC}{4}$, find the ratio AB: BC.

Answer: ______________________

6. If the third and fourth terms of an arithmetic progression are increased by 2 and 7 respectively, then the first four terms form a geometric progression. Which term of the arithmetic progression is 2008?

Answer: ______________________

7. Let $M_1 = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix}$, $M_2 = \begin{vmatrix} 9 & 11 & 13 \\ 15 & 17 & 19 \\ 21 & 23 & 25 \end{vmatrix}$, $M_3 = \begin{vmatrix} 27 & 29 & 31 & 33 \\ 35 & 37 & 39 & 41 \\ 43 & 45 & 47 & 49 \\ 51 & 53 & 55 & 57 \end{vmatrix}$, … .

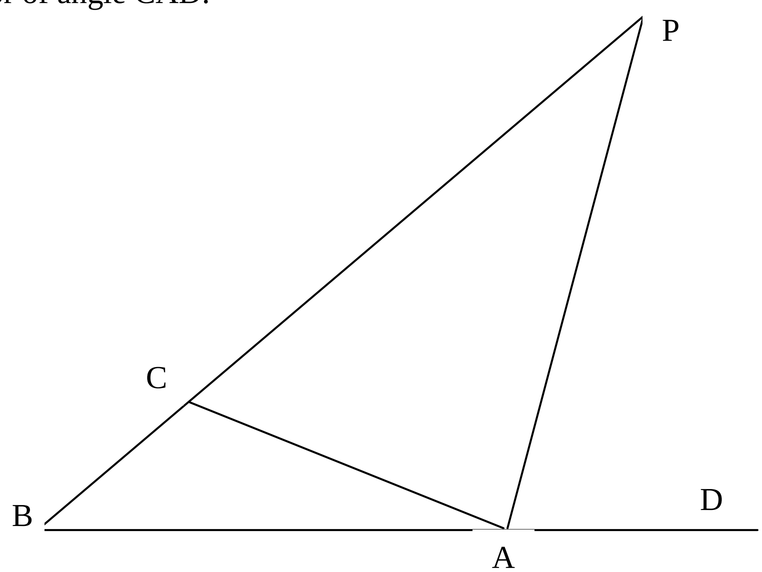
What are a) the smallest entry, and b) the largest entry in M_{25}?

Answer: a) _____________, b) _____________

8. Pentagon BCDEF is inscribed in a circle. The lengths of the sides are BC = CD = DE =EF = 8 and FB = 3. For triangles T_1 = BDF and T_2 = BCD, and their areas A_1 and A_2, respectively, find the ratio $A_1 : A_2$.

Answer: ______________________

164. In triangle ABC, AC = 7 and AB = 8. The side BC is extended, as pictured, to create the line segment BP. The line segment PA is the bisector of angle CAD. Compute the ratio BC : PC.



A For a particular real number a, the function f(x) is defined by f(x) = $|3x - a|$.
Using function f(x) = $|3x - a|$, the equation f(f(x)) = x has solutions whose sum is 27. Find the number a.

B.. The sequence t_1, t_2, t_3, t_4, …, t_{100} is defined by
$t_1 = 2$, $t_2 = 22$, $t_3 = 222$, $t_4 = 2222$, etc.
When the sum $t_1 + t_2 + t_3 + t_4 + \dots + t_{100}$ has last three digits *htu*, where *u* is the units digit of the sum. Find *htu*.

7. The number 3 is one root of the equation $x^6 - x^5 - 4x^4 + 5x^3 - 41x^2 + bx - 36 = 0$.
Find the other five roots.