VERMONT STATE MATHEMATICS COALITION TALENT SEARCH January 7, 2008

Test 3 of the 2007 - 2008 school year (*Test 4 will arrive at schools on February 18, 2008*)

PRINT NAME: _____ Sign Name: _____

School _____Grade _____

Math Department Head _____

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by February 4, 2008 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. For Coalition information and a copy of the test:
http://www.vtmathcoalition.org /

1. P is an interior point of the square ABCD for which PD = 10 and AP = CP = $10\sqrt{5}$.
Find the area of the square.

Answer:_____

2. A natural number *b*, all of whose digits are non-zero, satisfies the equation $b \cdot b' = 1000 + P(b)$, where b' is the number *b* with its digits reversed, and $P(b)$ is the product of the digits of *b*. Find all possible solutions for *b*.

Answer:_____

3. Find the smallest positive integer *n* for which n^{16} exceeds 16^{18}.

Answer:_____

4. The triangle ABC is a right triangle with legs BC = 3 and AC = 4. The two angle trisectors of angle C intersect the hypotenuse. The longer of these trisectors has length $\frac{a\sqrt{3}+b}{c}$, where gcf(a, b, c) = 1.
Evaluate $15a - 6b + 32c$.

Answer:_____

5. The positive integers a, b, and c satisfy the equations $\begin{aligned} a+b+c &= 2007 \\ a \cdot b - c &= 2007 \end{aligned}$
If c is a palindrome, find c.

Answer:_____

6. Find the smallest real number x, correct to the nearest hundredth, which satisfies the equation

$$(\log_2 x)^3 - \log_2 (2x^3) = (\log_2 x)^2 - \log_2 (x^2) - \log_2 2$$

Answer:_____

7. Let $w \neq 1$ be a root of the equation $x^3 = 1$. For the given real numbers A and B, find in simplest form the cubic equation whose roots are $A + B$, $Aw + Bw^{-1}$, and $Aw^2 + Bw^{-2}$.

Answer:_____

8. Let r, s, t, and u be the roots of the quartic equation $x^4 + kx + 3 = 0$, where r and s are negative integers and t and u are imaginary numbers. Write a quartic equation which has the four roots $\frac{r+s+t}{u^2}$, $\frac{r+s+u}{t^2}$, $\frac{r+t+u}{s^2}$, and $\frac{s+t+u}{r^2}$.

Answer:_____