VERMONT STATE MATHEMATICS COALITION TALENT SEARCH February 18, 2008

Test 4 of the 2007 - 2008 school year (This Test 4 concludes testing for the 2007-2008 year.)

PRINT NAME: __________________________ Sign Name: ___________________________

School ______________________________Grade _______

Math Department Head _____________________________

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by March 17, 2008 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. For Coalition information and a copy of the test:
http://www.vtmathcoalition.org /

1. In this problem x, y, z, and w are integers and $w \geq 0$. You are asked to find the smallest value of x for which $\begin{cases} y = x - 2008 \\ z = 2y - 2008 \\ w = 4z - 2008 \end{cases}$

Answer:___________________________

2. In the equations $f(x) = |3x - a|$ and $f(f(x)) = x$, both x and a represent real numbers. If the sum of the x-values satisfying the pair of equations is 27, find the value of a.

Answer:___________________________

3. You are given the equation $1 + 2 + 3 + \ldots + n = (n+1) + (n+2) + (n+3) + \ldots + (n + k)$.

a) Find the smallest positive integers n and k which satisfy the equation.

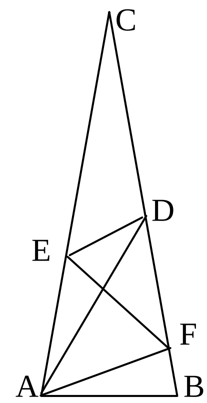
Answer: n = _______and k = _________

b) Find the next smallest positive integers n and k which satisfy the equation.

Answer: n = _______and k = _________

4. Refer to the pictured isosceles triangle in which AC = BC, AD = CD, and AB = AE = AF = EF.

Find the degree measure of $\angle ADE$.



Answer:____________

5. The function f has as its domain the set of ordered pairs of positive integers. Function f satisfies the following properties:

a) $f(x, x) = x$

b) $f(x, y) = f(y, x)$

c) $(x + y) \times f(x, y) = y \times f(x, x + y)$

Evaluate f(18, 50).

Answer:______________________________

6. In an arithmetic progression $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$, the twentieth term $a_{20} = \log(20) = \log_{10}(20)$ and the 32^{nd} term $a_{32} = \log(32) = \log_{10}(32)$. One term of the arithmetic progression is a rational number k. a) Solve for n, and b) solve for k for which $a_n = k$.

Answer a): ____________and b) ___________

7. Consider the three simultaneous equations $\begin{cases} |x| - y - z = 5 \\ x - |y| + z = -9 \\ x - y + |z| = -1 \end{cases}$. Find the unique solution of the simultaneous equations in which x, y, and z are all integers.

Answer:______________________________

8. In trapezoid ABCD, AB $||$ CD, $\angle A$ is a right angle, AB = 4, AD = 17, CD = 12, and point E lies on AD such that the measure of $\angle AEB$ is half the measure of $\angle CED$. Find the ratio AE : ED.

Answer:______________________________