Vertical State Mathematics Coalition Talent Search October 1, 2008

Test 1 of the 2008 – 2009 school year

PRINT NAME:__________________________ Signature:______________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade_____________

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by October 27, 2008 and submitted to:

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1. Find all the points of intersection of the surfaces whose equations are as follows:

$$z^2 = 2xy - 100 \quad \text{and} \quad z = \frac{x^2 + 2y^2 - 100}{2y}$$

Answer:____________________

2. The Vermont Tennis Club invites 64 players of equal ability to compete in a single elimination tournament (a player losing a match is eliminated). What is the probability that a particular pair of players (for example, Mike and Bob) will play each other at some point during the tournament?

Answer:____________________

3. Find two 2 digit integers such that the greater is 3 more than three times the smaller, and their sum is the reverse of the smaller number.

Answer:____________________

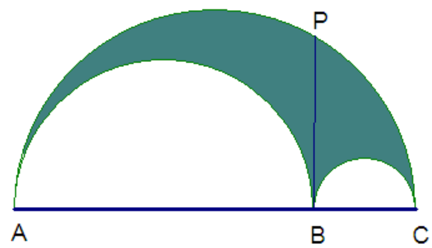
4. Find *all* ordered pairs (a,b) which satisfy $5y^2 + 2xy - 80 = 0$ such that (a,b) are both integers.

Answer:____________________

5. A right triangle has legs of length $\sqrt{22}$ and ($6+2\sqrt{10}$), find the *simplest possible* expression for the length of the hypotenuse. *Hint: Your answer should be a sum of whole multiples of square roots of whole numbers.*

Answer:____________________

6. In the following diagram, line segment **AC** is the diameter of a semicircle. For an arbitrary point B on segment **AC,** construct semicircles with segments **AB** and **BC** as diameters. Let ***h*** denote the length of segment **PB**, where P is a point on the original semicircle with segment **PB** perpendicular to segment **AC**. Express the area of the shaded region in terms of ***h*.**



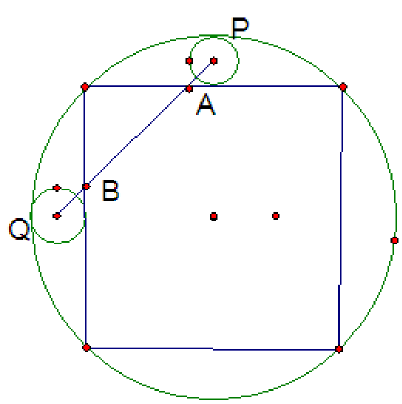
Answer:____________________

7. The first term of an arithmetic progression (AP) is -1 and the 8th term of a geometric progression (GP) is 80. The 4th terms of both progressions are equal. If the 6th term of the GP is equal to the sum of the 6th and 7th terms of the AP, find the smallest possible integral value of the 9th term of the GP.

Answer:____________________

8. A square is inscribed in a circle of radius 1. Circles P and Q are the largest circles which can be inscribed in the indicated segments of the circle. The line joining the centers of circles P and Q intersects the square at points A and B. Compute the length of AB.

Answer:____________________



′ The Math Coalition is grateful for problem contributors for this test including Middlebury College professors Michael Olinick, Bill Peterson , Peter Schumer and Frank Swenton. Also contributing is Tony Trono, retired Burlington High School math teacher.