Vermont State Mathematics Coalition Talent Search January 5, 2009

Test 3 of the 2008 – 2009 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by January 23, 2009 and submitted to:

Barbara Unger
Vermont State Math Coalition
735 Quaker Village Road
Weybridge, VT 05753

1. Find the area of the region bounded by the graphs of the following inequalities.

$$2x+y-3>0$$
$$x-2y+1<0$$
$$x+y-5<0$$
$$y-4<0$$

Answer_____

2. A product of three consecutive integers has the form $abcabc$. Find all sets of three consecutive integers and their ($abcabc$) products.

Answer_____

3. $f(x)$ is a linear function. If, for all real $\boldsymbol{x}$, $f\left(f\left(f(x)\right)\right)=f(x)$ find all solutions for $f(x)$.

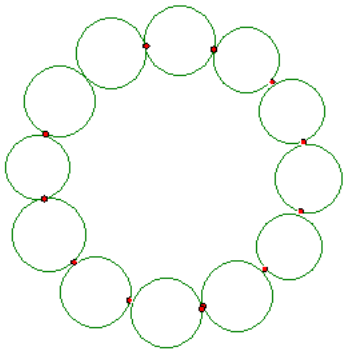
Answer_____

4. The product of two of the four roots of the following quartic equation is 8.
Find the value of $\boldsymbol{k}$.

$$x^4-256x^3+kx^2-494x-2008=0$$

Answer:_____

5. Given twelve unit circles arranged symmetrically into a ring, with each circle tangent to its two neighbors, as below. Find the radius of the circle circumscribing this ring (i.e., containing the twelve unit circles and tangent to each one).
[Tip: your answer should be a sum of whole numbers and square roots of whole numbers.]



Answer:_____

6. Consider the integers from 1 to 10,000,000. Calculate the number of integers that have the digit 1 in their representation.

Answer:_____

7. Evaluate the following: $\sum_{k=1}^{n} \frac{2k}{k^4+k^2+1}$

Answer:_____

8. A triangle has side lengths of 1, 1 and $\sqrt{3}$. Three mutually externally tangent circles are then drawn, each centered at one of the three vertices of the triangle. Find the fraction of the triangles area that lies within the circles.

Answer:_____

The Math Coalition is grateful for problem contributors for this test including Middlebury College professors Michael Olinick, Bill Peterson , Peter Schumer and Frank Swenton. Also contributing is Tony Trono, retired Burlington High School math teacher and Evan Dummit a mathematics student at the California Institute of Technology.