Vertmont State Mathematics Coalition Talent Search February 9, 2009

Test 4 of the 2008 – 2009 school year

PRINT NAME:__________________________ Signature:______________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade_____________

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by March 14, 2009 and submitted to:

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1. A student performing arithmetic is given the fraction 16/64 to reduce into lowest terms. He accidentally cancels the two sixes to obtain 1/4. Miraculously, he still obtained the correct answer! Find all other triples of nonzero, distinct base-ten digits (*a,b,c*) such that 'wrongly' reducing the fraction *ab/bc* to *a/c* will still yield the correct answer.

Answer_________________________

2. The sum of 19 consecutive positive integers equals p^3 where p is a prime number. Compute the smallest of the 19 integers.

Answer:____________________

3. A group of 5 scientists are working on a secret project, for which their materials are kept in a safe. They want to be able to open the safe only when a majority of the group is present. Therefore, the safe is provided with ***m*** different locks, and each scientist is given keys to exactly ***k*** of these locks. What are the minimum values of ***m*** and ***k*** for which such a scheme can be implemented?

Answer:____________________

4. Given $f(x) = \log_2 x$ and $g(x) = 2^x$, find $\boldsymbol{x}$ if $f\left(g(x)^{-1}\right) + g(-f(x)) = -1$

Answer:____________________

5. Let A and B be two points in the plane. Construct, successively circle C_1 with center A and radius equal to the length of AB. Let D be the other intersection point of C_1 and the line AB (i.e. the one that is not at B). Second, construct circle C_2 with center B and radius equal to the distance BD, and let E be the other intersection point of C_2 and the line AB (i.e., the one that is not at D). Third, construct circle C_3 with center E and radius equal to the distance AE, and let F be one of the intersection points of C_3 and C_1. Finally, construct circle C_4 with center F and radius equal to the distance AF, and let G be the other intersection point of C_4 and the line AB (i.e., the one that is not at A). Find the ratio of the length AG to the length AB.

Answer:____________________

6. Find all four-digit numbers $\boldsymbol{n}$ such that $\boldsymbol{n}$ is equal to 13 times a number resulting from removing one digit from $\boldsymbol{n}$. (Note: four-digit numbers do not start with 0.)

Answers:____

7. In a unit square ABCD, a circular arc S with center at A, passes through adjacent vertices B and D. Three circles, with centers at K, H and G and radii r, r_1 and r_2 are located as follows. Circle K is tangent to S and sides BC and CD. Circle H is tangent S, circle K and CD and circle G is tangent to S, circle H and CD. Find r_1 and r_2.

Answer:__ r_1=__________________

Answer:__ r_2=__________________

8. Given quadrilateral ABCD, with BC||AD, AB=BC=CD=5. Find the maximum area of the quadrilateral.

Answer:____________________

The Math Coalition is grateful for problem contributors for this test including Middlebury College professors Michael Olinick, Bill Peterson, Peter Schumer and Frank Swenton. Also contributing is Tony Trono, retired Burlington High School math teacher and Evan Dummit a mathematics student at the California Institute of Technology.