Vermont State Mathematics Coalition Talent Search September 28, 2009

Test 1 of the 2009 – 2010 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by October 19, 2009 and submitted to:

Barbara Unger
Vermont State Math Coalition
735 Quaker Village Road
Weybridge, VT 05753

Problem 1.

The fraction $\frac{2}{x^2-3x+2}$ can be written as an infinite series. Find the sum of the first four terms of the series expansion for $x=-1$ and $x=-2$

Answers:
when $x=-1$ _____

when $x=-2$ _____

Problem 2.

Many different seven digit integers can be formed by permuting the digits 1- 7; examples include 1236574 and 7531246. Suppose we list every possible seven digit number of this form and then add up all these numbers. What is the sum?

Answer: _____

Problem 3.

Two sides of an isosceles triangle have lengths 181 and 38. What is the area of the triangle?

Answer: _____

Problem 4.

In baseball's World Series, two teams play games against each other until one of them has won four games. (Thus the series must end no later than the seventh game.) Suppose that the teams are perfectly matched, so that each has a 50% chance of winning any game, and successive games are independent.
Under these assumptions, is it more likely the World Series will end in six games or in seven games? *Explain your answer. No credit without explanation*

Answer: _____

Problem 5.
In triangle ABC, angle B is obtuse and $AB > BC$. The bisector of the exterior angle at A meets the extension of BC at a point D. The bisector of the exterior angle at B meets the extension of AC at a point E. If $AB = AD = BE$, find the measure of angle ADB.

Answer: _____

Problem 6.
Given $f(x) + f(y) = f(x+y) - xy - 1$ and $f(1) = 1$, find another integer N such that $f(N) = N$.

Answer: _____

Problem 7.
Find the area of the largest equilateral triangle that can fit within a unit square.

Answer: _____

Problem 8.

All numbers in this problem are written in base seven. Find the base seven ordered triple (x, y, z) satisfying the following system.

$$11x - 5y - 12z = 21 \quad (1)$$
$$10x + 11y + 3z = 15 \quad (2)$$
$$22x + 15y \quad\quad = 22 \quad (3)$$

Answer: _____

The Math Coalition is grateful for problem contributors for this test including Middlebury College professors Michael Olinick, Bill Peterson, Peter Schumer and Frank Swenton. Also contributing is Tony Trono, retired Burlington High School math teacher and Evan Dummit a graduate mathematics student at the California Institute of Technology.