Vertmont State Mathematics Coalition Talent Search November 2, 2009

Test 2 of the 2009 – 2010 school year

PRINT NAME:_________________________ Signature:_____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by November 23, 2009 and submitted to:

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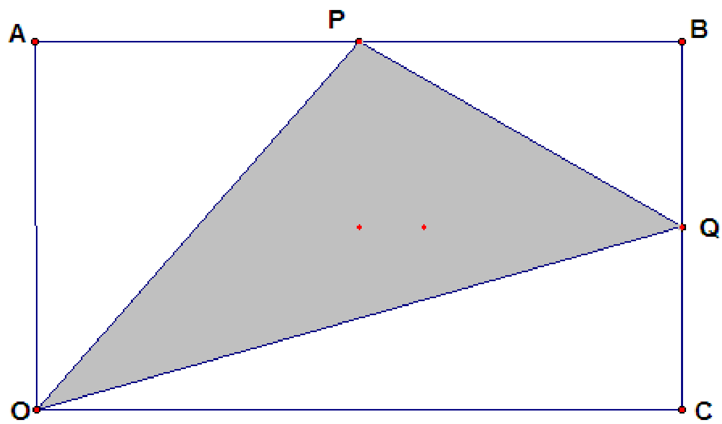
Problem 1.

Find the smallest positive integer which is evenly divisible by 225 and whose digits are all zeros or ones.

Answer: _______________

Problem 2.

You are given a rectangle OABC from which you remove three right-angled triangles, leaving a fourth triangle OPQ as shaded in the diagram below.



How must you position the points P and Q so that the area of each of the three removed triangles is the same? That is, what are the ratios PB : PA and QB : QC?

Answer: _______________

Problem 3.
Line A is drawn parallel to diagonal BD of square ABCD. A circle with radius BD and center B intersects line A at point E. Find angle ABE.

Answer: ________________

Problem 4.
The repeating decimal $x = 0.\overline{8}$, $y = 0.\overline{81}$ and $z = 0.\overline{814}$ have a product which is a repeating decimal $0.\overline{ace}$. Find the sum of $a, c,$ and e.

Answer: ________________

Problem 5.
Find the area of the region that lies under the graph of $f(x) = ||||6 - x| - x| - x| - x|$ and above the x axis, between $x = 0$ and $x = 12$.

Answer: ________________

Problem 6.
Alex is playing a card game and tabulating the results. He calculates that he has played 1800 games and won exactly 1542 of them. Rounded to the nearest percent, this is 86%. What is the smallest number of consecutive games he would have to win in order for his winning percentage (rounded to the nearest percent) to be equal to 87%?

Answer: ________________

Problem 7.
Find the smallest positive x which satisfies the inequality $x(x+1)(x+2)(x+3) \geq \dfrac{9}{16}$.

Answer: ________________

Problem 8.
Consider the four numbers $x, y, z,$ and w. The first three are in arithmetic progression and the last three are in geometric progression. If $x + w = 16$ and $y + z = 8$, evaluate $2009(x + y) + 50z + 95w$.

Answer: ________________

Special Note:
1. **Students are asked to send their email address to barbara.unger735@gmail.com**
2. **The third test will be available on February 8, 2010 at www.vtmathcoalition.org**

The Math Coalition is grateful for problem contributors for this test including Middlebury College professors Michael Olinick, Bill Peterson, and Peter Schumer. Also contributing is Tony Trono, retired Burlington High School math teacher and Evan Dummit a graduate mathematics student at the California Institute of Technology.