Vermont State Mathematics Coalition Talent Search February 8, 2010

Test 3 of the 2009 – 2010 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by March 3, 2010 and submitted to:

Barbara Unger
Vermont State Math Coalition
1043 Topelis Drive
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Problem 1.
Find all real values of x satisfying $x^{2\log_2 x} = 8$.

Answer: _____

Problem 2.
A natural number x, all of whose digits are nonzero, satisfies the equation $x \cdot \overline{x} = 1000 + p(x)$, where $\overline{x}$ is the number x with its digits reversed, and $p(x)$ is the product of the digits of x. Find all possible numbers x.

Answer: _____

Problem 3.
A line from ***a*** to ***b*** has midpoint at ***c.*** A point is chosen at random on the line and marked ***x***. Find the probability that the line segments *ax, bx*, and *ac* can be joined to form a triangle.

Answer: _____

Problem 4.
Find the smallest natural number ***n*** for which $\sum_{i=1}^{n} i = \sum_{i=1}^{k} (n+i)$ for $k > 1$.

Answer: _____

Problem 5.
Triangle ABC is a right triangle with legs $BC = 3$ and $AC = 4$. The length of the longer angle trisector from C to the hypotenuse is $\frac{a\sqrt{3}+b}{c}$. Evaluate $11a-10b+3c$.

Answer: _____

Problem 6.
Let the digital sum of a number be defined as the base 10 sum of its digits. Thus 400_{10} has a digital sum of 4, but $400_{10} = 294_{12}$ has a digital sum of 15 and $400_{10} = 110010000_2$ has a digital sum of 3. Certain numbers, when written is base 4, have a digital sum of 17. Let the digital sum of such a number, when the number is written is base 2, be K. If $\boldsymbol{x}$ and $\boldsymbol{y}$ are the minimum and maximum possible values of K respectively, compute the ordered pair $(\boldsymbol{x},\boldsymbol{y})$

Answer: _____

Problem 7.
At a high school in Germany, 80% of all students in the school are male, but only 5% of the male students and 10% of the female students are on the math team. If a member of the math team is selected at random, with each member of the team having an equal chance of being selected, compute the probability that the selected member is male

Answer: _____

Problem 8.
What is the largest integer multiple of 8, no two of whose digits are the same?

Answer: _____

Special Note:

1. **The fourth test will be available on March 10, 2010 at www.vtmathcoalition.org**
2. **Students should provide their email address below:**

Email Address: _____

The Math Coalition is grateful for problem contributors for this test including Middlebury College professors Michael Olinick, Bill Peterson, and Peter Schumer. Also contributing is Tony Trono, retired Burlington High School math teacher and Evan Dummit a graduate mathematics student at the California Institute of Technology.