Vermont State Mathematics Coalition Talent Search   March 10, 2010

Test 4 of the 2009 – 2010 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by March 31, 2010 and submitted to:

Problem 1.
A very productive farmer finds that he has 7^{2009} eggs. He packs them into cartons each holding a dozen eggs, until he no longer has enough eggs to fill a carton. Then he takes the leftover eggs and makes an omelet. How many eggs go into his omelet?

Answer: _____

Problem 2.
Make two random marks on a long stick.
(a) If you then break the stick into k pieces of equal length, what's the chance the two marks are on the same piece?
(b) If you then break the stick into k pieces of random lengths, what's the chance the two marks are on the same piece?

Answer a: _____

Answer b: _____

Problem 3.
. Find the sum of the solutions of $x^{1/4} = \dfrac{12}{7 - x^{1/4}}$

Answer: _____

Problem 4.
Leonhard has ten rods having lengths 1, 2, ... ,10 respectively. How many different ways are there to make a triangle by choosing three appropriate rods?

Answer: _____

Problem 5.
A point is chosen randomly inside a square of side length 5, and a unit circle is drawn with that point as its center. Calculate the probability that the circle does not intersect either of the square's diagonals or any of its sides.
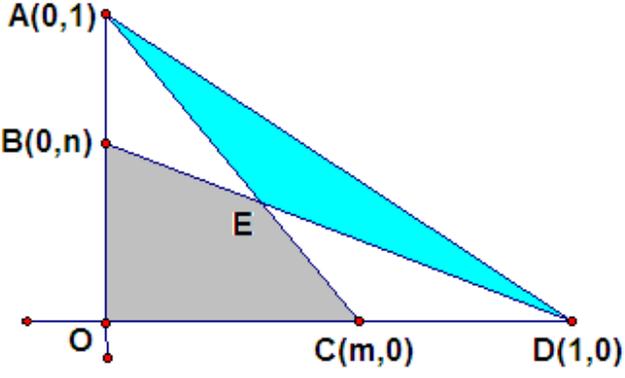
Answer: _____

Problem 6.
Circles A, B and C with radii 2, 4, and 6 respectively are tangent to one another. The common external tangent to circles A and B intersects the common external tangent to circles A and C at point x. Find the measure of angle x.

Answer: _____

Problem 7.

Let the area of $\triangle AED = R$ and the area of $OCEB = S$. For $0 < m, n < 1$ express $|R - S|$ in terms of m and n.

Answer: _____

Problem 8.
In the complex plane, let u and v be two distinct solution of $z^{2009} - 1 = 0$. Find the probability that $|u + v| \geq 1$.

Answer: _____

Special Note:
1. This tests solutions will be available on April 7, 2010 at www.vtmathcoalition.org
2. Students should provide their email address below:

    Email Address: _____