Vermont State Mathematics Coalition Talent Search October 13, 2010

Test 1 of the 2010 – 2011 school year

PRINT NAME:__________________________ Signature:______________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade_____________

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by November 03, 2010 and submitted to:

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Problem 1.

Let **Ş** be the set of all ordered triples of rational numbers and let the operation Ω be defined as follows: $(a,b,c)\Omega(d,e,f)=(ad,bd+ce,cf)$. Find the inverse of $\left(4,-5,\frac{1}{6}\right)$ for the operation Ω.

Answer: ________________

Problem 2.

In a sequence of 200 numbers, every one (except the end numbers) is equal to the sum of the two adjacent numbers in the sequence. The sum of all 200 numbers is equal to the sum of the first 100 numbers. The 32^{nd} number is equal to 32. If the sequence is continued, find the sum of the 2010^{th} and 2011^{th} numbers.

Answer: 2010^{th} ________________

2011^{th}_________________

Problem 3.

The hypotenuse of each of four right triangles is equal to ***h***. The sum of the lengths of the shorter legs when taken three at a time is 89, 100, 104 and 127. The ratio of the areas of the two smaller triangles is $\frac{13}{33}$. Find the sum of the areas of the four triangles.

Answer: ________________

The Vermont Math Coalition is grateful to problem contributors for this test including Tony Trono, retired Burlington High School Math teacher and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI.

Problem 4.
In 3D space, the points $A=(a,0,0)$, $B=(0,b,0)$, $C=(0,0,c)$ lie on a XYZ coordinate axis where the origin is designated $O=(0,0,0)$. If the areas of $\Delta ABO=12$, $\Delta BCO=4$ and $\Delta ACO=6$ find the area of ΔABC.

Answer: _______________

Problem 5.
The following products are equal: $9(REDSOX)=4(SOXRED)$.
Let T be the sum of the 6 digit numbers $REDSOX$ and $SOXRED$.
If $T+1=b^n$, find the ordered pair (b,n).

Answer: _______________

Problem 6.
A set S is called 'special" if, for any two distinct elements x and y in S, the element $x+y$ is not in S. For example $\{1,3,5\}$ is special, but $\{1,3,4\}$ is not special. Find the largest integer n such that the set $\{2,3,4,...n\}$ may be written as the union of two special subsets.

Answer: _______________

Problem 7.
A 55 gallon fuel container is mistakenly filled with fuel containing 6% ethanol. How many gallons must be removed and then replaced with a 50% ethanol mixture in order that the resulting fuel solution is 10% ethanol.

Answer: _______________

Problem 8.
In acute triangle ABC, altitudes BP and CQ are drawn with P on AC and Q on AB. If $CP=2012$ and $BC=2515$, find the sine of angle BQP.

Answer: _______________

Note: Test 2 will be available at
http://www.vtmathcoalition.org/talent-search/
on November 17, 2010.

To receive the next tests via email, clearly print your email address below:

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