Vertmont State Mathematics Coalition Talent Search February 2, 2011

Test 3 of the 2010 – 2011 school year

PRINT NAME:_________________________ Signature:_____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked no later than March 02, 2011 and submitted to:

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Problem 1.
Find the sum of all angles θ, $0 \le \theta \le 2\pi$, such that
$\left(8\cos 4\theta - 3\right)\left(\cot\theta + \tan\theta - 2\right)\left(\cot\theta + \tan\theta + 2\right) = 12$

Answer: ________________

Problem 2.
Four circles are drawn such that each circle is tangent to the other three circles. The radii of the three larger circles are 1, 2 and 3. Find the smallest possible radius of the fourth circle. The four circles cannot be tangent at the same point.

Answer: ________________

Problem 3.
In a narrow alley near UVM, a ladder leans against a wall at an angle of 75^{o} with the horizontal ground and reaches a point m feet above the ground. Keeping the foot of the ladder at the same point, the top of the ladder is moved to the wall on the other side of the alley. The ladder now makes an angle of 45^{o} with the horizontal and reaches a point n feet above ground $\left(m > n\right)$.

Find, *in simplest form,* the width of the alley in terms of m and n.

Answer: ________________

The Vermont Math Coalition is grateful to problem contributors for this test including Tony Trono, retired Burlington High School Math teacher and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI.

Problem 4.
For $x > 0$, and $(4x)^{\log 2} - (9x)^{\log 3} = 0$, find x.

Answer: ________________

Problem 5.

Evaluate: $2\sum_{k=1}^{k=1005}\left[(2k-1)(2k+1)\right]^{-1}$

Answer: ________________

Problem 6.
Three urns collectively contain 15 red marbles and 15 blue marbles. One marble is drawn randomly from each urn. If the probability that all three marbles are blue is $\frac{11}{125}$, what is the probability that all three marbles are red?

Answer: ________________

Problem 7.
Find the sum of n terms of an arithmetic progression whose first term is the sum of the first n positive integers and whose common difference is n.

Answer: ________________

Problem 8.
Let the three vertices of a triangle T be $(2,7),(4,-1)$ and $(0,y)$. Find y so that the perimeter of the triangle T is as small as possible.

Answer: ________________

Note: Test 4 will be available at
http://www.vtmathcoalition.org/talent-search/
on March 16, 2011.

To receive the next tests via email, clearly print your email address below:

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