Vermont State Mathematics Coalition Talent Search March 16, 2011

Test 4 of the 2010 – 2011 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by April 6, 2011 and submitted to:

Barbara Unger
Vermont State Math Coalition
1043 Topelis Drive
Englewood, FL 34223

Problem 1.
Find the area of a triangle whose medians have lengths of 39, 42 and 45.

Answer: _____

Problem 2.
In quadrilateral $ABCD$, $\cot A = 4$, $\cot B = \frac{3}{2}$, $\cot C = 5$. Find all possible values for $\cot D$.

Answer: _____

Problem 3.
In ΔABC, $AB = 4, BC = 5$ and $AC = 6$. Equilateral triangles ABD and CBF are drawn exterior to triangle ABC. CD and AF are drawn. Find the sum of $X + Y$ where $X = \angle ACD$ and $Y = \angle CAF$

Answer: _____

Problem 4.
Given $f(n) = \left(\frac{5+3\sqrt{5}}{10}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{5-3\sqrt{5}}{10}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$
If $f(n+1) - f(n-1) = kf(n)$ where k is an integer, find k.

Answer: _____

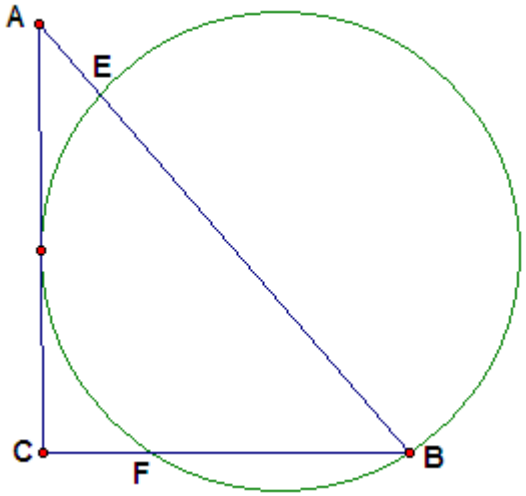
The Vermont Math Coalition is grateful to problem contributors for this test including Tony Trono, retired Burlington High School Math teacher and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI.

Problem 5.

A circle is tangent to leg AC of right ΔABC and intersects the hypotenuse AB at E and leg BC at F. Point B is on the circles circumference.

AC and BC have integral lengths and $AC > BC$.

If $AE = 4$ and $BE = 21$ find the radius of the circle.



Answer: _____

Problem 6.

Two numbers from the set $S = \{1, 2, 3, ..., 106\}$ are selected at random and multiplied. What is the probability that the product is a multiple of 5.

Answer: _____

Problem 7.

Let $a, b, c,$ and d be positive real numbers such that $\log_a b = c$, $\log_b c = 2d$, $\log_c d = 3a$, and $\log_d a = 4b$. Find the numerical value of the product $abcd$.

Answer: _____

Problem 8.

Find the sum $\sum_S \frac{1}{st}$ where S is the collection of all ordered pairs (s,t) of relatively prime positive integers such that $0 < s < t \le 12$ and $s + t > 12$.

Answer: _____

http://www.vtmathcoalition.org/talent-search/

The Vermont Math Coalition is grateful to problem contributors for this test including Tony Trono, retired Burlington High School Math teacher and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI.