Vermont State Mathematics Coalition Talent Search October 12, 2011

Test 1 of the 2011 – 2012 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by November 02, 2011 and submitted to:

Barbara Unger
Vermont State Math Coalition
735 Quaker Village Road
Weybridge, VT 05753

To receive the next tests via email, clearly print your email address below:

_____

Problem 1.

In ΔABC points P and Q are chosen on sides $\overline{AB}$ and $\overline{AC}$ respectively, and segment $\overline{PQ}$ is drawn meeting median $\overline{AM}$ at X. If $AP = \frac{1}{4} AB$ and $AQ = \frac{1}{2} AC$, find the ratio $\frac{PX}{PQ}$.

Answer: _____

Problem 2.

Ten points are chosen on a circle and all of the chords determined by these points are drawn.

(a) How many chords are there?

(b) Assume that no three of these chords intersect at a common point inside the circle. How many points inside the circle lie on two chords?

Answers: a) _____

b)_____

The Vermont Math Coalition is grateful to problem contributors for this test including Tony Trono, retired Burlington High School Math teacher and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI.

Problem 3.
In trapezoid *ABCD*, *AB* parallel to *CD*, angle A is a right angle and *AB* = 4, *AD* = 17, *CD* = 12, and *E* lies on *AD* such that $\angle AEB = \frac{1}{2}\angle CED$. Find the ratio $AE : ED$.

Answer: _____

Problem 4.
Evaluate the sum $1\cdot(1+\frac{1}{n})+2\cdot(1+\frac{1}{n})^2+3\cdot(1+\frac{1}{n})^3+\ldots+n\cdot(1+\frac{1}{n})^n$ in terms of *n*.

Answer: _____

Problem 5.
If *a, b* and *c* are real numbers and $a+b+c=16$, $c^a = b^{2a}$, $2^c = 2\cdot 4^a$ and $abc<0$
Evaluate $9a-6b+9c$.

Answer: _____

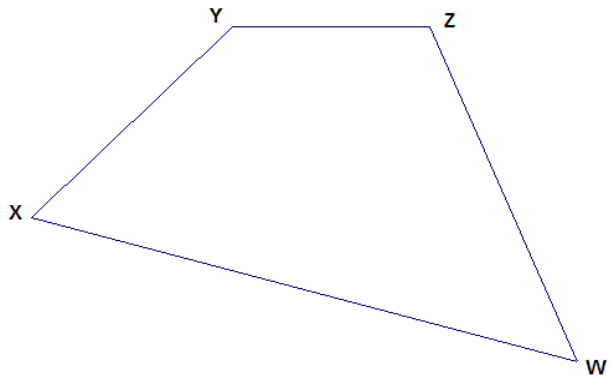
Problem 6.
If $a_n = 2-\frac{n-1}{2}\log_{10} 5$ and $b_n = 8\cdot 100^{a_n}$, evaluate $\sum_{n=1}^{\infty} b_n$.

Answer: _____

Problem 7.

In quadrilateral $wxyz$, angle ***y*** is 135°, angle ***z*** is 120°, $xy = 3\sqrt{6}$, $wz = 8$, $yz = 8-3\sqrt{3}$

Find the length of xw.



Answer: _____

Problem 8.
The *harmonic mean* of two positive real numbers *a* and *b* is the reciprocal of the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. A pair (a,b) of positive integers is called *dramatic* if the arithmetic mean of *a* and *b* equals 1 plus the harmonic mean of *a* and *b*.

If (a,b) is dramatic and $a \geq 2011$, find the minimal possible value for *b*.

Answer: _____

Note: Test 2 will be available at
http://www.vtmathcoalition.org/talent-search/
on November 16, 2011.

The Vermont Math Coalition is grateful to problem contributors for this test including Tony Trono, retired Burlington High School Math teacher and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI.