Vermont State Mathematics Coalition Talent Search   November 16, 2011

Test 2 of the 2011 – 2012 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by December 7, 2011 and submitted to:

> Barbara Unger
> Vermont State Math Coalition
> 1043 Topelis Drive
> Englewood, FL 34223

To receive the next tests via email, clearly print your email address below:

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Problem 1.

Find the asymptotes of $f(x) = \dfrac{3x^2 - 5x - 2}{x^2 + 5x - 14}$

Answer: _____

Problem 2.

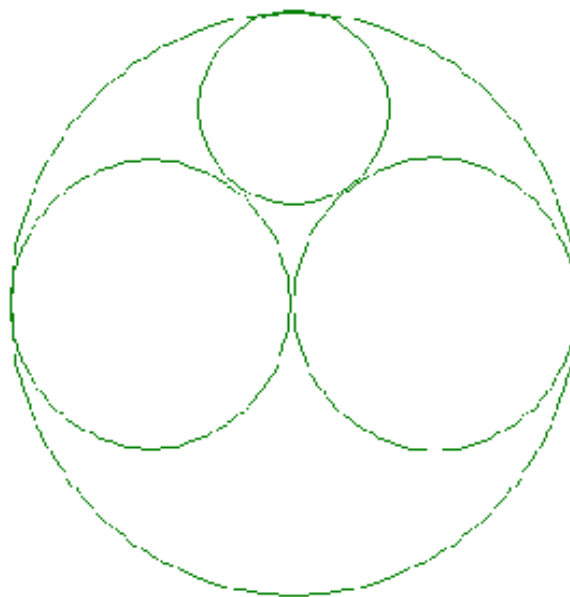Find all real numbers x such that $\sqrt[3]{x+4} - \sqrt[3]{x} = 1$

Answer: _____

Problem 3.

Suppose $a_0, a_1, a_2, ..., a_n$ are positive real numbers satisfying $a_i a_{n-i} = 1$ for all $i = 1, 2, ..., n$.

If k is any integer, compute the sum $\dfrac{1}{1+a_0^k} + \dfrac{1}{1+a_1^k} + \dfrac{1}{1+a_2^k} + ... + \dfrac{1}{1+a_n^k}$

Answer: _____

Problem 4.

Shown here are four circles each tangent
to the other three. The largest of these
has radius 2 and each of the medium
sized circles has radius 1.

Find the radius of the smallest circle

Answer: _____

Problem 5.

Millie the cat wants to put on her socks and shoes. Her cabinet contains four identical
socks and four identical shoes, and she draws them out one at a time in a random order. If
she draws a sock, she puts it on one of her bare feet, and if she draws a shoe she will put
it on a foot that has a sock, unless none of her feet have a sock, in which case she gives
up. What is the probability that she is able to put on all of her socks and shoes without
giving up?

Answer: _____

Problem 6.

Let $x_n = \dfrac{\log_2(n^{\sqrt{n}}) \cdot \log_3(n^{\sqrt{n}})}{\log_3(n) + \log_{\sqrt{2}}(n)}$ for $2 \le n \le 7$. Find the number of positive integers which

divide $18^{x_2 + x_3 + x_4 + x_5 + x_6 + x_7}$.

Answer: _____

Problem 7.

Evaluate the sum $\displaystyle\sum_{n=-2012}^{n=2012} \dfrac{1}{3^n + 1}$

Answer: _____

Problem 8.

There is a unique rational number p/q, with $q < 1,000,000$, such that the decimal
expansion of p/q begins with 0.01040916253649. Find the ordered pair (p, q) .

Answer: _____