Vermont State Mathematics Coalition Talent Search February 1, 2012

Test 3 of the 2011 – 2012 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by February 29, 2012 and submitted to:

Barbara Unger
Vermont State Math Coalition
1043 Topelis Drive
Englewood, FL 34223

To receive the next tests via email, clearly print your email address below:

_____

Problem 1.
Three circles are drawn inside another circle such that each of the four circles is tangent to the other three circles. Two circles with radii *a* and *b*, where $a > b$, have their centers on the diameter of the largest circle.

a) Find the radius of the largest circle in terms of *a* and *b*.
b) For what natural numbers *a* and *b* will the radius of the smallest circle be closest to one.

Answer: a)_____

Answer: b)_____

Problem 2.
Let a, b, c and d be the distinct roots of $p(x) = x^4 + 2x^3 - 4x^2 - x + 1$. The six values $ab, ac, ad, bc, bd,$ and cd are roots of the polynomial *q(x)*. If $q(0) = 1$, find $q(1)$.

Answer: _____

Problem 3.
If $f(x) = \dfrac{3x+5}{3-5x}$, find all *x* for which $f(x^{-1}) = f^{-1}(x)$.

Answer: _____

The Vermont Math Coalition is grateful to problem contributors for this test including Tony Trono, retired Burlington High School Math teacher and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI.

Problem 4.
Kathleen, Robert, Mary and Jordan are playing a game with a standard 6 sided die. They each roll the die, and the person who rolls the lowest number wins. If there is a tie, the people who tied each roll again and the lowest new roll wins. If there is still a tie, they continue rolling until someone wins the tiebreak. Jordan rolls first and gets a 3. What is the probability that he will win?

Answer: _____

Problem 5.
Three circles are drawn inside another circle such that each of the four circles is tangent to the other three circles. If the radii of the three inner circles are 1, 2, and 3, find the radius of the largest circle.

Answer: _____

Problem 6.
In algebra class, Yvonne was copying a graphing calculator problem from the board, but her teacher abruptly erased the problem before Yvonne finished writing. The portion of the problem that Yvonne was able to copy reads "Find the three values of x at which the graph of the line $y = 2x+1$ is tangent to the graph of $y = x^6 - 4x^5 + 2x^4 + 8x^3 - "$.
Given that Yvonne only missed the quadratic, linear and constant terms of the function, what is the answer to Yvonne's problem?

Answers: _____

Problem 7.
If $\sum_{n=1}^{99} \frac{1}{n(n+1)} = K$ and $KQ = 495$, find Q.

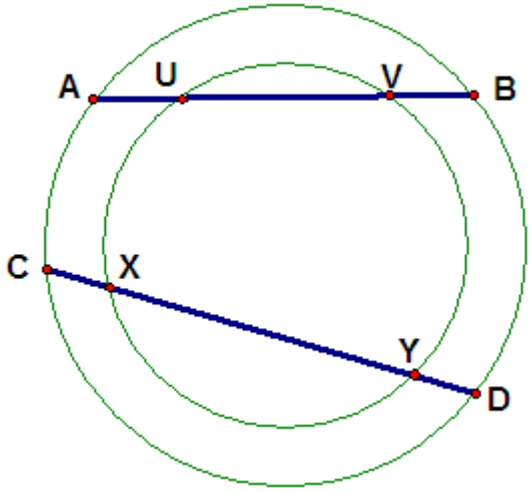
Answer: _____

Problem 8.

Suppose chords AB and CD of a circle meet a smaller concentric circle at points U, V, X and Y.

If $AU = 2, UV = 10$ and $CX = 3$ find XY

Answer: _____



Note: Test 4 will be available at
http://www.vtmathcoalition.org/talent-search/
on March 14, 2012

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