Vermont State Mathematics Coalition Talent Search March 16, 2012

Test 4 of the 2011 – 2012 school year

PRINT NAME:__________________________ Signature:______________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by April 6 , 2012 and submitted to:

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Problem 1.
Triangle ABC has $AB = 10$ and $12BC = 13AC$. Find the maximum area for ΔABC .

Answer: ________________

Problem 2.
Everyone has just one “magic birthday”, when his age is exactly equal to the sum of the digits of the year of his birth. For example, the magic birthday of someone born in 1899 was in 1926. Notice that someone born in 1908 also had a magic birthday in 1926. Find the next year after 1926 in which two people born in different years can both have magic birthdays.

Answer: ________________

Problem 3.
Find all 3-tuples of complex numbers (x, y, z) satisfying the following system of equations:

$$(x^2 + xz)(y^2 + yz) = 36$$
$$(x^2 + xy)(z^2 + yz) = 225$$
$$(y^2 + xy)(z^2 + xz) = 100$$

Answer: ________________

The Vermont Math Coalition is grateful to problem contributors for this test including Tony Trono, retired Burlington High School Math teacher and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI.

Problem 4.

The equation $2\cdot\left[\sqrt{x}-\cfrac{1}{\sqrt{x}-\cfrac{1}{\sqrt{x}-\cfrac{1}{\sqrt{x}-\ldots}}}\right]^2 = x+\cfrac{1}{x+\cfrac{1}{x+\ldots}}$ has a unique real positive solution for *x*. Given that $x \cong 4.969$, find *x* exactly in the form $\frac{a\sqrt{b}+c}{d}$ where a, b, c, d are integers.

Answer: _______________

Problem 5.

An ordered 3-tuple (a,b,c) of positive integers is called a *cubic triple* if $\sqrt[3]{a+b\sqrt{c}}+\sqrt[3]{a-b\sqrt{c}}=1$. For example, $(2,1,5)$ is a cubic triple because $\sqrt[3]{2+\sqrt{5}}=\frac{1+\sqrt{5}}{2}$ and $\sqrt[3]{2-\sqrt{5}}=\frac{1-\sqrt{5}}{2}$ and their sum is obviously one. There exists a unique family of cubic triples given (given in terms of the parameter *t*) as $(a,b,c)=(\alpha_1 t+2,\alpha_2 t+1,\alpha_3 t+5)$ where α_1,α_2 and α_3 are pairwise coprime positive integers. Find the sum $\alpha_1+\alpha_2+\alpha_3$.

Answer: _______________

Problem 6.

At the Mathville post office, the largest shipping container is a rectangle measuring 27 klogs by 22 klogs. Ivan has been commissioned to make a rectangular painting which he must ship at the Mathville post office. The painting must be exactly 30 klogs long, but Ivan can choose the width. How wide can he make the painting (in klogs) and still fit it into a shipping container?

Answers: _______________

Problem 7.

Find $\left\lfloor \log_2\left(\frac{255!}{3!253!}+\frac{255!}{5!251!}+\frac{255!}{7!249!}+\ldots+\frac{255!}{253!3!}\right)\right\rfloor$ where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to ***x***.

Answer: _______________

Problem 8.

Find the sum of all possible digits *a* with the property that some cube of an integer ends in the digits *aaa* (in base 10).

Answer: _______________

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