Vertical State Mathematics Coalition Talent Search October 4, 2012
Test 1 of the 2012 – 2013 school year

PRINT NAME:________________________ Signature:____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions may be emailed to johlson@sbschools.net or be postmarked by November 01, 2012 and submitted to:

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To receive the next tests via email, clearly print your email address below:

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Problem 1.

A 4 x 4 *antimagic* square is an arrangement of the numbers from 1 to 16 (inclusive) in a square, so that the totals of each of the four rows and four columns and two main diagonals are ten consecutive numbers in some order. The diagram below shows an incomplete antimagic square. When it is completed what number will replace the star?

4	5	7	14
6	13	3	*
11	12	9	
10			

Answer: ____________

Problem 2.

Suppose $p(x) = x^2 + cx + d$ satisfies $p(c) = p(d) = c.$ If $cd \neq 0,$ find p(2).

Answer: ____________

The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson, Dr. Robert Poodiack and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI. With additional support from Tony Trono a retired math teacher from Burlington High School.

Problem 3.

A cube of side 2 is inscribed in a sphere. The sphere is inscribed in a cone with slant height equal to the diameter of its base. The cone is inscribed in a right circular cylinder. What is the surface area of the cylinder (including top and bottom)?

Answer: ____________

Problem 4.

ABCD is a trapezoid with AB || CD. Diagonals AC and BD intersect at k. The line through k parallel to AB intersects AD and BC at P and Q respectively. If AB = 8 and CD = 12, find the length of PQ.

Answer: ____________

Problem 5.

A die is tossed. If the die shows a 1 or a 2 then one coin is tossed. If the die shows a 3 then two coins are tossed. Otherwise, three coins are tossed. Given that the resulting coin toss (es) produced no heads, what is the probability that the die showed a 1 or a 2?

Answer: ____________

Problem 6.

Define the "star product" (ф) of two numbers as the sum of the product of the corresponding digits. So 246 ф 738 = 2 x 7 + 4 x 3 + 6 x 8 = 74. Find the sum of the digits of M and N so that M ф N = 602 and M + N is a minimum.

Answer: ____________

Problem 7.

Find all positive integers *n* that are within 201.3 of exactly 15 perfect squares. *Note*: The perfect squares are {0, 1, 4, 9 …}. The within is not strict, that is *n* is within *r* of *m* if the difference between *n* and *m* is less than **or equal** to r. (e.g. 300 is within 100 of 200).

Answer: ____________

Problem 8.

A real number is called "integrally radical" if it is the sum of some number of terms of the form $\sqrt{k}$ and $-\sqrt{k}$, where *k* is a positive integer. For example, $3 = \sqrt{9}, \sqrt{2} - \sqrt{3}$, and $\frac{6}{\sqrt{2}} = \sqrt{2} + \sqrt{2} + \sqrt{2}$ are integrally radical, but $\frac{1}{3}, \frac{\sqrt{3}}{2}, and\ \frac{5}{\sqrt{2}}$ are not. Find the smallest positive integer *n* such that $\frac{n}{\sqrt{5}+\sqrt{6}+\sqrt{7}}$ is integrally radical.

Answer: ____________

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