Vermont State Mathematics Coalition Talent Search November 13, 2012
Test 2 of the 2012 – 2013 school year

PRINT NAME:________________________ Signature:____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions may be emailed to johlson@sbschools.net or be postmarked by December 11, 2012 and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

____

Problem 1.

An ultraprime is a prime number all of whose (base-10) digits are prime numbers, and such that each of the integers obtained by rearranging its digits is also a prime number. Find the number of ultraprimes less than 1000.

Answer: ____________

Problem 2.

In a mathematical competition, a contestant can score 5, 4, 3, 2, 1, or 0 points for each problem. Find the number of ways a contestant can score a total of 30 points for 7 problems.

Answer: ____________

Problem 3.

Find the total area enclosed by the graph of $|3x + y| + |x - 2y| = 14$.

Answer: ____________

The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson, Robert Poodniak and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI. With additional support from Tony Trono a retired math teacher from Burlington High School.

Problem 4.

Rods 1 meter in length are used to build a rigid cubic framework. Twelve rods are needed to build a cube of side 1 meter. By fitting together eight of these unit cubes, a cubic framework can be constructed that has side 2 meters. By using 27 of the unit cubes, a cubic framework can be constructed that has side 3 meters. Each time the unit cubes are combined, there are duplicate rods along the edges where the cubes fit together. The duplicates may be removed (leaving one rod where there were previously two) and re-used elsewhere. What is the minimum necessary side length of the cubic structure such that the total length of all the rods used would stretch the 384,000 km from the Earth to the Moon?

Answer: ____________

Problem 5.

Suppose $x^2 + y^2 = 14x + 6y + 6$. What is the maximum value of 3x + 4y?

Answer: ____________

Problem 6.

In triangle ABC, AB = 3, BC = 4, and AC = 6. D is the intersection of the angle bisector of A with BC, E is the intersection of the angle bisector of B with AC, and F is the intersection of the angle bisector of C with AB. Find the ratio of the area of triangle DEF to the area of triangle ABC.

Answer: ____________

Problem 7.

How many integers between 1,000,000 and 10,000,000 have digits which do not decrease in order from left to right? (1234567 and 1122599 would be such integers, but 1204567 and 1122595 would not).

Answer: ____________

Problem 8.

The function $f(x)$ has the properties that (i) $f\left(\frac{x}{x+1}\right) = \frac{1}{2}\,f(x)$, and (ii) $f(1-x) = 1 - f(x)$, for all positive real numbers x. Find;

$$\sum_{k=1}^{\infty} f\left(\frac{1}{k}\right)$$

Answer: ____________