

Vermont State Mathematics Coalition Talent Search March 12, 2013
Test 4 of the 2012 – 2013 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions may be emailed to joholson@sbschools.net or be postmarked by April 9, 2013 and submitted to:

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Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

Problem 1.

Find all real numbers x such that $(x^2 - x - 1)^{x^3 + 2x^2 - 9x - 18} = 1$. Answer: _____

Problem 2.

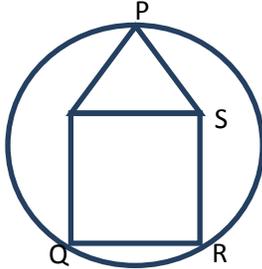
There are 100 people in a room. Each person states his/her birth date (month and day, excluding February 29). Let N be the number of different birth dates. What is the expected value of N to the nearest integer? (Assume that the 365 birth dates are equally likely.)

Answer: _____

The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson, Robert Poodiack and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI. With additional support from Tony Trono a retired math teacher from Burlington High School.

Problem 3.

An equilateral triangle sits atop a square as in the diagram. All sides have length 1. A circle passes through points P, Q and R. What is the radius of the circle?



Answer: _____

Problem 4.

A set of positive integers is defined to be *wicked* if it contains no three consecutive integers. We count the empty set, which contains no elements at all, as a *wicked* set. Find the number of *wicked* subsets of the set $\{1,2,3,4,5,6,7,8,9,10\}$.

Answer: _____

Problem 5.

The “polar midpoint” of a segment whose endpoints are $A = (r_1, \theta_1)$ and $B = (r_2, \theta_2)$ when written in polar coordinates (with $0 \leq \theta_1, \theta_2 < 2\pi$) is the point $C = (\frac{r_1+r_2}{2}, \frac{\theta_1+\theta_2}{2})$ in polar coordinates. Find the square of the distance between the (standard) midpoint and the polar midpoint of the segment whose coordinates in polar are $(4\sqrt{2}, \frac{\pi}{12})$ and $(8, \frac{7\pi}{12})$.

Answer: _____

Problem 6.

A Morgan horse farmer willed that, upon his death, his three sons would receive the u -th, v -th, and w -th parts of his horses, respectively, where u , v , and w are integers. The farmer had N horses in the herd when he died. The three sons, however, could not evenly divide the team of horses according to the requirements, so they asked a Talent Search competitor for help. She rode over on her own horse, which she added to the team. It was then possible to divide up the new team of horses into 4 parts: one u -th of the horses, one v -th of the horses, one w -th of the horses, and the Talent Search competitor's original horse.

Find all solutions (u, v, w, N) .

Answer: _____

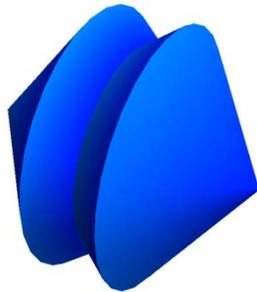
Problem 7.

Suppose that x , y , and z are positive real numbers (none of which equals 1) such that $\log_x yz + \log_y z = 5$ and $\log_z xy + \log_y x = 3$. Find all possible real numbers α such that $yz = x^\alpha$.

Answer: _____

Problem 8.

A certain credit card has the shape of a rectangle of dimensions 3 units x 4 units. If you rotate the card about one of its diagonals, what is the volume of the resulting solid? Here is a picture of the solid.



Answer: _____