

Vermont State Mathematics Coalition Talent Search September 18, 2013

Test 1 of the 2013 – 2014 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions may be emailed to joholson@sbschools.net or be postmarked by October 19, 2013 and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

Problem 1.

In the array, two letters are called neighboring letters if they are adjacent to each other horizontally, vertically, or diagonally. Starting from any letter "M" on the outside of the array find the number of ways of spelling "MATH" by moving only between neighboring letters.

Answer: _____

M	M	M	M	M	M	M
M	A	A	A	A	A	M
M	A	T	T	T	A	M
M	A	T	H	T	A	M
M	A	T	T	T	A	M
M	A	A	A	A	A	M
M	M	M	M	M	M	M

Problem 2.

Find the number of positive integers n such that n plus the product of the (base-10) digits of n equals 2014.

Answer: _____

Problem 3.

The Calgary Lames and the Boston Ruins play a best-of-7 series with no ties, in which the series ends once one team has won exactly 4 games. The probability that the Ruins win any particular game is constant, and the games are independent. If it is exactly 45% more likely that the series lasts for 6 games than for 7 games, find the probability that it lasts for 4 games.

Answer: _____

Problem 4.

a) Find all $x > 0$ such that $x + \frac{1}{x + \frac{1}{x + \frac{1}{x + \dots}}} = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}}$ Answer: _____

b) Find all $x > 0$ such that $x - \frac{1}{x - \frac{1}{x - \frac{1}{x - \dots}}} = \sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - \dots}}}}$ Answer: _____

Problem 5.

Find all pairs of non-zero real numbers (x, y) that are solutions to the simultaneous equations $x^{x+y} = y^3$ and $y^{x+y} = x^6 y^3$.

Answer: _____

Problem 6.

Inside a circle of radius 5 is inscribed a trapezoid of height 4, one of whose bases is a diameter of the circle. A triangle having the property that each of its sides is parallel to a side of the trapezoid is also inscribed in the circle. Find the area of the triangle.

Answer: _____

Problem 7.

Find the number of ordered pairs of real number (x, y) satisfying the equations $x^2 + y^2 = 2013$ and $(\tan \pi x)(\tan \pi y) = 1$.

Answer: _____

Problem 8.

One angle in a triangle with sides $a - 3$, $a - 2$, and $a - 1$ is supplementary to one angle in a triangle with sides $a - 1$, a , $a + 1$. Find all possible values of a .

Answer: _____