

Vermont State Mathematics Coalition Talent Search November 1, 2013

Test 2 of the 2013 – 2014 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions may be emailed to joholson@sbschools.net or be postmarked by December 2, 2013 and submitted to:

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Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

Problem 1.

A survey of 350 math students was taken to see which Star Trek Captain was their favorite. The captains listed were Kirk, Picard and Archer.

- 46 students liked none of the captains listed (maybe preferring Sisko and Janeway)
- $\frac{1}{3}$ of the students who liked Archer also liked Picard but not Kirk
- 98 students only liked Kirk
- 14 students liked all three captains
- 184 students liked Picard
- 56 students liked both Kirk and Picard
- 82 students liked more than 1 captain

Find the number of students who liked;

a) Only Archer

Answers: a) _____

b) Only Picard

b) _____

c) Kirk and Archer

c) _____

Problem 2.

Triangle ABC has $\angle BAC = 90^\circ$. The perpendiculars from A to the internal bisectors of $\angle ABC$ and $\angle ACB$ intersect said bisectors at R and P , respectively. Determine the measure of $\angle RAP$.

Answer: _____

Problem 3.

The positive integer X has 4026 digits, all of which are 4's. The positive integer Y has 2013 digits, all of which are 8's. What is the sum of the digits of $\sqrt{X - Y}$?

Answer: _____

Problem 4.

A check is cashed at a bank and mistakenly the teller pays out the number of cents as dollars and the number of dollars as cents. The person receiving the money spends \$3.50 before noticing the mistake, and on counting the remaining money finds it is exactly double the amount of the original check. What was the original check amount?

Answer: _____

Problem 5.

Three positive integers are randomly chosen, with replacement, from the set $\{1, 2, \dots, 2013\}$. Let t be the expected value of the smallest of the three integers. Find the integer closest to t .

Answer: _____

Problem 6.

In the multiplication problem below, each letter represents a different digit: Which digit does C represent?

$$\begin{array}{r}
 A\ B\ C\ D\ E\ F\ G\ H \\
 \times \qquad \qquad \qquad A\ J \\
 \hline
 C\ C\ C\ C\ C\ C\ C\ C\ C
 \end{array}$$

Answer: _____

Problem 7.

Triangle ABC is inscribed in circle O . Points D and E are chosen on AB and BC respectively, such that $AC = BD = 4$, $AD = BE = 2$, and $BC = 3$. Segment DE is extended to intersect arc AB at F and arc BC at G . Find $|DF - EG|$.

Answer: _____

Problem 8.

Find the value of the infinite product $3 \cdot 3^{\log_4 3} \cdot 3^{\log_4 3^{\log_4 3}} \cdot 3^{\log_4 3^{\log_4 3^{\log_4 3}}} \dots$. Express your answer in the form $a^{\log_b c}$ for rational numbers a , b , and c .

Answer: _____