**Vermont State Mathematics Coalition Talent Search** January 2, 2014
Test 3 of the 2013 – 2014 school year

PRINT NAME:________________________ Signature:____________________
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____________________________________________ Grade___________

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions may be emailed to johlson@sbschools.net or be postmarked by January 30, 2014 and submitted to:

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Vermont State Math Coalition
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**To receive the next tests via email, clearly print your email address below:**

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**Problem 1.**

The unit squares in a 3 x 3 grid are colored blue and gray at random, and each color is equally likely. What is the probability that a 2 x 2 square will be blue?

Answer: ____________

**Problem 2.**

Find the sum of the absolute values of the six distinct values for $\alpha$ such that $(\alpha^3 - 9\alpha^2)^2 = 8$.

Answer: ____________

**Problem 3.**

In triangle ABC, the trisectors of angle C divide AB into segments of lengths 4,6, and 8 (in some order). Find all possible values of AC + BC.

Answer: ____________

The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson, Robert Poodiack and Evan Dummit .

**Problem 4.**

Find the number of palindromes which have 16 digits, in which the product of the non-zero digits is 16, and the sum of the digits is also equal to 16. How many such numbers are there?

Answer: ___________

**Problem 5.**

If $\log_9 A = \log_{18} B = \log_{16} C = 2013^{2014}$, find $\log_C \left(\frac{B}{A}\right)$.

Answer: _____________

**Problem 6.**

Let $PQRS$ be a unit square. Define $A, B, C$ and $D$ to be points on $PQ, QR, RS$ and $SP$ respectively, such that $PA = QB = RC = SD = \frac{1999}{2000}$. Construct the triangles, $QDR, RAS, SBP$ and $PCQ$. Find the area of the region which is common to all four triangles.

Answer: _____________

**Problem 7.**

If $f(x)$ is a function, its "first twist" is defined to be $f_1(x) = \frac{f(x+f(x))}{f(x)}$, and then its "$n$th twist" $f_n(x)$ is defined recursively to be the first twist of its $(n-1)st$ twist $f_{n-1}(x)$. For example, the first twist of $f(x) = 3x$ is $f_1(x) = \frac{3(x+3x)}{3x} = 4$ and the second twist is $f_2 = \frac{4}{4} = 1$. Let $p(x) = x^2 - 20x + 13$, and let $p_n(x)$ be the $n$th twist of $p(x)$. Find the smallest value of $n$ such that all zeroes of $p_n(x)$ are negative.

Answer: _____________

**Problem 8.**

A finite collection of positive integers has arithmetic mean 10. Find all possible integral values for the geometric mean of the integers, if

a) there are no duplicates allowed in the collection. Answer: _________________

b) duplicates are allowed in the collection. Answer: _________________