

**Vermont State Mathematics Coalition Talent Search** September 23, 2014  
 Test 1 of the 2014 – 2015 school year

PRINT NAME: \_\_\_\_\_ Signature: \_\_\_\_\_

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School \_\_\_\_\_ Grade \_\_\_\_\_

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlnson@sbschools.net or be postmarked by October 17, 2014 and submitted to:

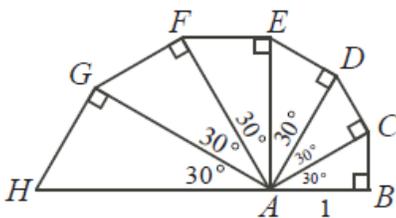
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To receive the next tests via email, clearly print your email address below:

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**Problem 1.**

A shell is formed from six triangular sections, as shown. Each triangle has interior angles of  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$ . If AB has a length of 1 cm, what is the length of AH, in cm?



Answer: \_\_\_\_\_

**Problem 2.**

Find all integer solutions of the system of simultaneous equations:

$$ab + cd = -1$$

$$ac + bd = -1$$

$$ad + bc = -1$$

Answer: \_\_\_\_\_

**Problem 3.**

Two teams play a 7-game series, where the first team to win 4 games wins the series. If a team has won  $w$  games and lost  $l$  games so far in the series, its probability of winning the next game is  $\frac{(w+1)}{(l+w+2)}$ .

Find the probability that the series requires 7 games to decide the winner.

Answer: \_\_\_\_\_

**Problem 4.**

In triangle  $ABC$ ,  $\overline{AB} = 4$ ,  $\overline{AC} = 5$ ,  $\overline{BC} = 6$ , and point  $D$  lies on  $\overline{AC}$  with  $\overline{CD} = 2$ . Points  $E$  and  $F$  on  $\overline{AB}$  and  $\overline{BC}$  (respectively) are chosen so that when line segments  $\overline{DE}$  and  $\overline{DF}$  are drawn, they divide  $ABC$  into three regions of equal area. Find the length of segment  $\overline{EF}$ .

Answer: \_\_\_\_\_

**Problem 5.**

Determine all real solutions to the system of equations.

$$x + \log(x) = y - 1$$

$$y + \log(y - 1) = z - 1$$

$$z + \log(z - 2) = x + 2$$

Answer: \_\_\_\_\_

**Problem 6.**

The natural numbers  $a, b, c$  have the property that  $a^3$  is divisible by  $b$ ;  $b^3$  is divisible by  $c$ ; and  $c^3$  is divisible by  $a$ . Prove that  $(a + b + c)^{13}$  is divisible by  $abc$ , but that  $(a + b + c)^{12}$  need not be divisible by  $abc$ .

*Note: For this problem, please include your proof on a separate sheet of paper.*