**Vermont State Mathematics Coalition Talent Search** November 3, 2014
Test 2 of the 2014 – 2015 school year

PRINT NAME:_________________________ Signature:_____________________
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School______________________________________________ Grade____________

Current Mathematics Teacher: ________________________________________________

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by December 1, 2014 and submitted to:


Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445


**To receive the next tests via email, clearly print your email address below:**

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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**Problem 1.**

Three numbers form a geometric sequence. The arithmetic mean of the first two is -9, and the arithmetic mean of the first and third terms is -15. Find the smallest possible value of the first term.

Answer: ____________

**Problem 2.**

A circular sector has the same perimeter and area as a rectangle. (The perimeter of the sector includes the two radii.) Prove that the radius of the sector equals one of the side lengths of the rectangle.

*Note: For this problem, please include your proof on a separate sheet of paper.*

**Problem 3.**

If $Q$ is a convex quadrilateral whose four side lengths and two diagonal lengths all lie in the set $\{1, d\}$, where $d > 1$, find all possible values of $d$.

Answer: ____________

**Problem 4.**

The remainder when dividing the polynomial $p(x)$ by $x^3 - 2x^2 - x + 2$ is $ax^2 + 6x + 12$. The remainder when dividing $p(x)$ by $x^3 + x^2 - 4x - 4$ is $3x^2 - bx - 12$. Find $a + b$.

Answer: __________

**Problem 5.**

The "run-length" of a sequence of heads and tails is the length of the longest consecutive sequence of identical outcomes: thus, the run-length of the sequence HTTHHHHTH is 4, while the run-length of the sequence TTTHHTHHTTT is 3. If a fair coin is flipped 12 times, find the probability that the run-length of the sequence of outcomes is 2.

Answer: __________

**Problem 6.**

Suppose that $1 < a < b < c < d < e$ are positive integers such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} = 1$. If $e$ is equal to 3 times a prime number, find the sum of all possible values for $d$.

Answer: __________

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