Vertical State Mathematics Coalition Talent Search February 16, 2015
Test 4 of the 2014 – 2015 school year

PRINT NAME:________________________ Signature:____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by March 10, 2015 and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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Problem 1.

Points $A, B, C, and\ D$ lie on circle O. Point F lies on CD such that OF is perpendicular to CD, and point E is the intersection of OF with AB. If $AB = 8, CD = 6, EF = 1$, and OF is perpendicular to AB, find the radius of circle O.

Answer: ____________

Problem 2.

Suppose $a, b, and\ c$ are integers greater than 1 such that $ab - c < 20$ and $ac - b < 15$. Find the maximum possible value for $bc - a$.

Answer: ____________

Problem 3.

Find $\tan^{-1}\left[\sqrt{\frac{2\sin(20^o)+\sin(33^o)+\sin(7^o)}{2\sin(20^o)-\sin(33^o)-\sin(7^o)}}\right]$. Express your answer in simplest form, in degrees.

Answer: ____________

Problem 4.

A dart is thrown at a triangular dartboard with vertices $(-1,3)$, $(0,-3)$, and $(2,2)$, and its landing location is randomly and uniformly distributed on the dartboard. Find the probability that the dart is closer to the origin than to any of the three vertices of the dartboard.

Answer: ____________

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Problem 5.

We say that a positive integer k is "3-special" if it has a divisor that is a distance of less than 3 from $\sqrt{k}$. For example, 2016 is 3-special: 42 divides 2016, and the distance between 42 and $\sqrt{2016} \approx 44.90$ is less than 3. Prove that there exist seven polynomials $p_1(n), p_2(n), \dots, p_7(n)$ such that an integer $k \geq 200$ is 3-special if and only if there exist some integers i and n with $1 \leq i \leq 7$ and $k = p_i(n)$.

Note: For this problem, please include your proof on a separate sheet of paper.

Problem 6.

Ten standard six-sided dice are rolled. The probability that no subset of six of these dice has a sum divisible by 6 is $\frac{N}{6^{10}}$. Compute the value of N.

Answer: ____________

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