Vermont State Mathematics Coalition Talent Search September 21, 2015
Test 1of the 2015 – 2016 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by **October 16, 2015** and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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1) In the following correctly-worked multiplication problem, all of the digits except for the 5s have been erased and some have been replaced with letters. Each letter represents a different digit 0-9 (other than 5) , every letter represents the same value everywhere it appears, and none of the integers starts with the digit 0. Find the value of the 8-digit number that is the result of the multiplication problem.

				F	I	V	E	
				F	I	V	E	
								F
+							I	
+		5		5		V		
+					E			
=			5					

Answer: _____

2) A point P is selected at random from the interior of a right triangle with legs 3 and 4. Find the probability that the sum of the squares of the distances from P to the three vertices of the triangle is less than 18.

Answer: _____

3) Find all ordered pairs of integers (x, y) such that;

$$x^3 = 89x + 40y$$
$$y^3 = 40x + 89y$$

Answer: _____

4) In triangle ABC, $\tan(A) = 2\tan(B) = 3\tan(C)$. Compute $\cos A \cdot \cos B \cdot \cos C$.

Answer: _____

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5) Find all irrational numbers α such that $\alpha^2 - 2\alpha$ and $\alpha^3 - 6\alpha$ are both rational numbers.

Answer: _____

6) For fixed positive integers n, a, and b the game of Extreme Hopscotch is played on a line of $n + 1$ tiles labeled with the integers 0 through n inclusive. Grace starts at tile 0 and makes a sequence of hops: in each hop, she may either move a tiles forward or b tiles backward (but is not allowed to hop out of the line of tiles). To win the game, Grace must make a sequence of hops starting at tile 0, reaching tile n, and then returning back to tile 0. For example, if $n = 5, a = 2$, and $b = 3$, Grace could win via the sequence $0,2,4,1,3,5,2,4,1,3,0$. If a and b are relatively prime positive integers, prove that Grace has a winning strategy if and only if $n \geq a + b - 1$.

Note: For this problem, please include your proof on a separate sheet of paper.

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