Vermont State Mathematics Coalition Talent Search November 2, 2015

Test 2 of the 2015 – 2016 school year

PRINT NAME:________________________ Signature:____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade___________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by **November 30, 2015** and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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1) Observe that 25 is a perfect square, and the integer obtained by increasing all of its digits by 1 (namely, 36) is also a perfect square. Find the next smallest integer possessing this property: namely, that it is a perfect square, and the integer obtained by increasing all of its digits by 1 is also a perfect square. *Note: it is not possible to increase by 1 the digits of any number containing a digit 9, because 9+1 is not a base-10 digit.*

Answer: ____________

2) In circle O, chords AB and CD intersect at E. If $AE = 3 \cdot BE$ and $CE = 12 \cdot DE$, find $\frac{AC}{BD} + \frac{BC}{AD}$.

Answer: ____________

3) A random function f from the set $S = \{1,2,3,4\}$ to the set $T = \{5,6,7\}$ is chosen, and a random function g from T to the set $U = \{8,9\}$ is chosen. Find the probability that the range of the composite function $g \circ f$ is U.

Answer: _____________

4) In rectangle $ABCD$, there exist points E on BC and F on CD such that triangle AEF is equilateral. If the area of AEF is $16\sqrt{3}$ and the area of $ABCD$ is $34\sqrt{3}$, find the area of triangle CEF.

Answer: ____________

5) An "odd Egyptian fraction decomposition into n terms" of a rational number $\frac{p}{q}$ is a representation $\frac{p}{q} = \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}$ where $a_1 < a_2 < \cdots < a_n$ are odd positive integers. For example, $\frac{3}{5} = \frac{1}{3} + \frac{1}{5} + \frac{1}{15}$ is an odd Egyptian fraction decomposition of $\frac{3}{5}$ into three terms.

a) Prove that there is no odd Egyptian fraction decomposition of 1 into 2016 terms.
b) Prove that there is an odd Egyptian fraction decomposition of 1 into 2015 terms.

Note: For this problem, please include your proof on a separate sheet of paper.

6) Let a and b be real numbers such that $a^4b^3 + a^3b^4 = 2160$ and $a^5b^2 + a^2b^5 = 29520$. Find the value of $(a + 7)(b + 7)$.

Answer: ____________

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