Vermont State Mathematics Coalition Talent Search January 4, 2016

Test 3 of the 2015 – 2016 school year

PRINT NAME:_____   Signature:_____

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by **January 29, 2016** and submitted to:

<div align="center">

Jean Ohlson

Vermont State Math Coalition

PO Box 384

Charlotte, VT 05445

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To receive the next tests via email, clearly print your email address below:

_____

1) Find all pairs of positive integers (x, y) such that $7x - 4y = 1$ and $y < x\sqrt{3}$.

Answer: _____

2) A (nondegenerate) triangle with side lengths $\cos\theta$, $\cos\theta$ and $2\sin\theta$ has area $\cos 2\theta$. Find the area of the triangle whose side lengths are $\cos^2\theta, \cos^2\theta$, and $2\sin^2\theta$.

Answer: _____

3) Let circle O have radius 5 with diameter $\overline{AE}$. Point F is outside circle 0 shuch that lines $\overline{FA}$ and $\overline{FE}$ intersect circle O at points B and D, respectively. If $FA = 10$ and $m\angle FAE = 30^o$, then the perimeter of quadrilateral $ABDE$ can be expressed as $a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6}$ where a, b, c and d are rational. Find $a + b + c + d$.

Answer: _____

4) Right triangle DEF has $E = (2,2)$, while D lies on the curve $y = x^2 - x$ and F lies on the curve $y = 3x - x^2$. If two vertices of ΔDEF have the same x-coordinate, find all possibilities for the area of ΔDEF.

Answer: _____

5) Find the number of integers n such that the sum of all the even divisors of n is 2016.

Answer: _____

6) Suppose a, b, and c are positive real numbers such that $\max(a, b, c) \le 4 \min(a, b, c)$. Prove that $2ab + 2ac + 2bc \ge a^2 + b^2 + c^2$, and determine when equality can occur.

Note: For this problem, please include your proof on a separate sheet of paper.