Vermont State Mathematics Coalition Talent Search February 15, 2016
Test 4 of the 2015 – 2016 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by **March 11, 2016** and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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1) Mika is building a garden inside a square plot of land of side length 5 meters. She walks around the perimeter of the plot and places five posts, equally spaced along the 20-meter boundary. Her garden is to be the area inside the pentagon formed by the posts. Find the maximum possible area of her garden, in square meters.

Answer: _____

2) For their investment club, Alice, Barry, Charlotte, and Duane each buy an integral number of shares of stock from one of four companies, and then sell their shares a year later. Each person only buys shares from one company, and no two people purchased from the same company. The four purchase prices were \$150, \$250, \$300, and \$350, while the four sell prices (not necessarily in the same order) were \$100, \$300, \$400, and \$450. Given that

(i) Alice bought 6 shares, the most of anyone,

(ii) Barry bought the same number of shares as Charlotte and Duane combined,

(iii) Alice, Charlotte, and Duane each made the same total (positive) profit, and

(iv) Barry invested the second most money, \$1400, while Charlotte invested the least,

determine the average sell price (per share) of Barry's and Charlotte's combined shares.

Answer: _____

3) Find all bases b such that the result of the of the base-b arithmetic problem $121_b \cdot 41_b - 215_b \cdot 23_b - 36_b$ is a perfect square. (Note: $b > 6$ because the digit 6 appears in the problem.)

Answer: _____

4) Find all real solutions to the equations;

$$\sqrt{x+y} + \sqrt{x-y} = 10$$
$$x^2 - y^2 - z^2 = 476$$
$$2^{(\log|y| - \log z)} = 1$$

Answer: _____

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5) The side lengths of a triangle are three consecutive positive integers and the largest angle in the triangle is twice the smallest one. Determine the side lengths of the triangle.

Answer: _____

6) A set of integers is called "3-squarefree" if it contains no 3-element subset the product of whose elements is a perfect square. For example, the set $\{1,2,3,4\}$ is 3-squarefree, since the possible products of a 3-element subset are 6, 8, 12, and 24, none of which is a square. However, the set $\{1,2,3,6\}$ is not 3-squarefree, since the subset $\{2,3,6\}$ has product a perfect square.

(a) Prove that there is no 3-squarefree subset of $\{1,2,3\cdots,60\}$ containing 45 elements.

(b) Prove that there is a 3-squarefree subset of $\{1,2,3,\cdots,60\}$ containing 30 elements.

Note: For this problem, please include your proof on a separate sheet of paper.

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