Vermont State Mathematics Coalition Talent Search September 12, 2016

Test 1of the 2016 – 2017 school year

PRINT NAME:_________________________ Signature:_____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by **October 10, 2016** and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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1) Jean is given a decimal number with three decimal places. She first rounds it to the nearest hundredth, then to the nearest tenth, then to the nearest integer. (As usual, the digits 0 - 4 round down and 5 - 9 round up.) If she finishes with the number 8, what is the positive difference between the largest and smallest possible numbers she could have started with?

Answer: ____________

2) The 2017 Vermont Mathematics All-Stars A team, consisting of 15 students and 2 coaches, stands in random order in a circle. Find the expected number of students standing between the two coaches (measured the shortest possible way around the circle).

Answer: ____________

3) Alice and Bob are playing a game: they begin with a pile of k stones, and alternate turns with Alice going first. On each turn, a player may remove n^2 stones for any positive integer n not divisible by 5. Thus, for example, if the pile currently contains 39 stones, a player could remove 1, 4, 9, 16, or 36 of them. The winner of the game is the person who takes the last stone. Determine the number of values of $k, 1 \leq k \leq 2017$, for which Alice has a winning strategy.

Answer: ____________

4) An *alternating-digit integer* is an integer larger than 9 having the form $ababa\cdots$ for some digits a and b. (We explicitly allow the possibility that $a = b$, that $a = 0$, or that $b = 0$.) The twin of the *n*-digit alternating-digit integer $ababa\cdots$ is the *n*-digit integer $babab\cdots$. For example, 1313, 111, 30303, and 61616 are alternating-digit integers whose twins are respectively 3131, 111, 03030, and 16161. If P is an alternating-digit integer with an odd number of digits, whose twin Q is less than P, prove that $P^2 - Q^2$ is also an alternating-digit integer.

Note: For this problem, please include your proof on a separate sheet of paper.

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5) A circular arc drawn inside a triangle is called a *bisector arc* if it is centered at a vertex of the triangle, the endpoints of the arc lie on the sides of the triangle containing that vertex, and the arc separates the triangle into two pieces of equal area. Triangle ABC has $AB = 2, BC = 4$, and $AC = 2\sqrt{3}$. Given that ΔABC has three bisector arcs, find (in simplest form) the product of the lengths of the shortest and longest such arcs.

Answer: ____________

6) Suppose *z* is a complex number such that $|z + 3| = 10$.

(a) Find the maximum possible value of $|z^2 + 16|$. Answer: ____________

(b) Find the minimum possible value of $|z^2 + 16|$. Answer: ____________

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