

**Vermont State Mathematics Coalition Talent Search** December 5, 2016  
Test 3 of the 2016 – 2017 school year

PRINT NAME: \_\_\_\_\_ Signature: \_\_\_\_\_

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School \_\_\_\_\_ Grade \_\_\_\_\_

Current Mathematics Teacher: \_\_\_\_\_

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to [joholson@sbschools.net](mailto:joholson@sbschools.net) or be postmarked by **January 6, 2017** and submitted to:

Jean Ohlson  
Vermont State Math Coalition  
PO Box 384  
Charlotte, VT 05445

**To receive the next tests via email, clearly print your email address below:**

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1) The odd numbers from 5 to 21 inclusive are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9, and 17 are placed as shown, what is the value of  $x$ ?

	5	
9		17
$x$		

Answer: \_\_\_\_\_

2) Regular decagon  $ABCDEFGHIJ$  is inscribed in circle  $O$ , which has radius 2. The point  $P$  is located one-third of the way from  $A$  to  $B$  along the arc  $AB$ . Compute the sum of the 10 squared lengths  $AP^2 + BP^2 + CP^2 + \dots + JP^2$ .

Answer: \_\_\_\_\_

3) Krysta has two fair dice that have special labels: her blue die is labeled with the six standard trigonometric functions (sine, cosine, tangent, secant, cosecant, cotangent) and her red die is labeled with the six angles  $30^\circ, 45^\circ, 60^\circ, 120^\circ, 135^\circ, 150^\circ$ . Krysta rolls the two dice once, and computes the value  $x$  obtained by evaluating the trigonometric function on the blue die at the angle on the red die. She rolls both dice again and obtains the value  $y$  in the same manner. Find the probability that  $x > y$ .

Answer: \_\_\_\_\_

4) Cyclic quadrilateral  $ABCD$  has distinct integer side lengths and a circle  $O$  inscribed in it. Find the minimal possible integer value for the area of  $ABCD$ .

Answer: \_\_\_\_\_

5) Let  $a, b$ , and  $c$  be the three positive real roots of the cubic polynomial  $p(x) = x^3 - 7x^2 + 11x - 4$ , and set  $r = \sqrt{a} + \sqrt{b} + \sqrt{c}$ . Find the unique ordered triple of integers  $(s, t, u)$  with the property that  $r^4 + sr^3 + tr^2 + ur$  is an integer.

Answer: \_\_\_\_\_

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6) For a positive integer  $n$ , let  $p(n)$  denote the product of the positive divisors of  $n$ : thus,  $p(6) = 36$ . Recursively define  $p^d(n) = p(p^{d-1}(n))$  for each  $d \geq 2$ , with  $p^1(n) = p(n)$ : thus,  $p^2(6) = 10077696$ . Prove that  $\frac{1}{2} \cdot 3^{2016} < \log_2(\log_6(p^{2017}(6))) < 3^{2016}$ . [Partial credit will be offered for proving weaker bounds.]

*Note: For this problem, please include your proof on a separate sheet of paper.*