Vermont State Mathematics Coalition Talent Search January 23, 2017
Test 4 of the 2016 – 2017 school year

PRINT NAME:__________________________ Signature:______________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade_____________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by **February 20, 2017** and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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1) Let ABCDEFGH be a regular octagon of side length 4. Squares *ABIJ* and *BCKL* are constructed so that I, J, K, and L all lie inside the octagon. Find the area of the region of overlap between squares ABIJ and BCKL.

Answer: ____________

2) If ABC is a nondegenerate, non-right triangle and $(1 - 2\tan(A))(1 - 2\tan(B)) = 5$, find all possible values of $\sin(C)$.

Answer: _____________

3) For any $n \geq 3$, an n –digit integer is called a "canyon integer" if there is an integer $k, 2 \leq k \leq n-1$, such that its first k digits form a strictly decreasing sequence, and its last $n-k+1$ digits form a strictly increasing sequence. For example, 543212345, 976124, and 302 are canyon integers, whereas 987, 55234, 82196, and 1358 are not canyon integers. Find the total number of canyon integers. [Note: integers do not start with the number 0.]

Answer: ____________

4) Find all integers *n* for which $n^3 - 10n^2 + 20n + 17$ is the cube of an integer.

Answer: ____________

5) Find the minimum value of $f(a, b) = [3\cos(a) - 4\cos(b) + 12]^2 + [3\sin(a) - 4\sin(b) + 5]^2$ as a and b range over all real numbers.

Answer: ____________

6) Let $p(t) = 2t^4 - t^3 - 4t^2 + t + 1$, and let the four real roots of $p(t)$ be $a_1 < a_2 < a_3 < a_4$. Also let $q(t) = 4t^4 - 41t^3 + t^2 + 364t + 181$, and let the four real roots of $q(t)$ be $b_1 < b_2 < b_3 < b_4$. Prove that the four points $(x, y) = (a_1, b_1), (a_2, b_2), (a_3, b_3)$, and (a_4, b_4) all lie on the parabola $y = x^2 + 4x + 1$.

Note: For this problem, please include your proof on a separate sheet of paper.

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