1. Nick has 2 identical cups: the first cup is full of water, while the second is empty. He pours half the water from the first into the second. Then, on transfer 2, he pours \( \frac{1}{3} \) of the water in the 2nd cup back into the first cup. He repeats this, alternating cups, pouring \( \frac{1}{i(i+1)} \) of the water in a cup back into the other cup on the \( i \)th transfer. What fraction of the water is in the first cup just after the 2018th transfer?

Answer: __________

2. Suppose you are a sportsbook taking 100 bets for who will win the Super Bowl. If each bettor picks exactly one team from the Vikings, Jaguars, Eagles and Patriots, how many different combinations of wagers are possible for the 100 bets you are taking? Here are some examples of possible combinations. You do not distinguish between individual bettors.

(i) \( V = 30, J = 15, E = 40, P = 15 \).
(ii) \( V = 8, J = 32, E = 60, P = 0 \).
(iii) \( V = 0, J = 0, E = 0, P = 100 \).

Answer: __________

3. What is the length of the shortest path AQRB in the plane, where \( A = (14, 12), B = (29, 1) \), Q lies on the y-axis and R lies on the x-axis?

Answer: __________

4. If \( 0 < \theta < \frac{\pi}{2} \) and \( \frac{1 + 2 \sin(\theta) + 3 \sin^2(\theta) + 4 \sin^3(\theta) + \ldots}{1 + 2 \cos(\theta) + 3 \cos^2(\theta) + 4 \cos^3(\theta) + \ldots} = \frac{4}{81} \),

find the value of \( 1 + 2 \tan(\theta) + 3 \tan^2(\theta) + 4 \tan^3(\theta) + \ldots \).

Answer: __________

5. Triangle ABC has AB = 22, AC = 24, and BC = 26, and line l bisects both the perimeter and area of ABC. If line l intersects triangle ABC in points D and E, find all possible lengths of segment DE.

Answer: ________________________________

6. Let \( p(x) \) be a monic polynomial with integer coefficients. We call a triangle \( p \)-special if its 3 vertices all have integer coordinates and lie on the graph of \( y = p(x) \), and we say the positive integer \( n \) is \( p \)-special if there is a \( p \)-special triangle whose area is \( n \).

a) Show that there exists a polynomial \( p \) of degree 3 such that the \( p \)-special integers are precisely the positive multiples of 3.

b) Determine, with proof, whether there exists a polynomial \( p \) of degree 3 such that every positive integer is \( p \)-special.

Note: For this problem, please include your proof on separate sheets of paper.

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