

Vermont State Mathematics Coalition Talent Search -- March 2018

Test 4 of the 2017-2018 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **April 9, 2018** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

1. Suppose that $p(x)$ is a polynomial all of whose coefficients are either 0 or 1. If $p(\sqrt{2}) = 20 + 18\sqrt{2}$, find the value of $p(2)$.

Answer: _____

2. The graph of the equation $47x + 43y - 3 = 2018$ is drawn on graph paper with each square representing one unit in each direction. The grid lines begin at the origin, and are vertical and horizontal. How many of the 1×1 squares have interiors lying entirely below the graph and entirely in the first quadrant?

Answer: _____

3. The inscribed circle of triangle UVM is tangent to UV at X, and its radius is 21. Given that $UX = 23$ and $XV = 27$, find the perimeter of the triangle.

Answer: _____

4. We say that a sequence of distinct positive integers a_1, a_2, \dots, a_n forms a VMTS 25th anniversary sequence if $(1 - \frac{1}{a_1})(1 - \frac{1}{a_2}) \dots (1 - \frac{1}{a_n}) = \frac{25}{2018}$.

- a) Show that there exists a VMTS 25th anniversary sequence with 81 terms.
- b) Show that there does not exist a VMTS 25th anniversary sequence with 80 terms.

Note: For this problem, please include your proof on separate sheets of paper.

5. Three data scientists eat lunch in the cafeteria every day. They arrive there independently, at random times uniformly chosen between 11:00 AM and 1:00 PM, and stay for exactly 20 minutes. What is the probability that all three data scientists are in the cafeteria simultaneously at some point?

Answer: _____

6. Given that there exist unique rational numbers (a, b, c) for which $\cos^7(\pi/7) = a \cos(\pi/7) + b \cos(3\pi/7) + c \cos(5\pi/7)$ find the ordered triple (a, b, c) .

Answer: _____