Vermont State Mathematics Coalition Talent Search -- September 2018
Test 1 of the 2018-2019 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School_____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **October 26, 2018** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

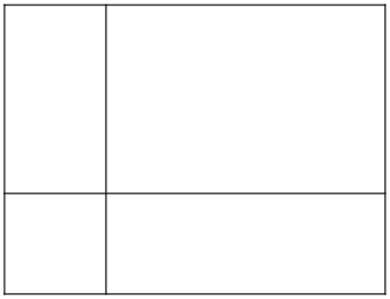
To receive the next tests via email, clearly print your email address below:

_____

The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Missisquoi Valley Union HS) and Evan Dummit (Postdoctoral Associate at Arizona State University).

Vermont State Mathematics Coalition Talent Search -- September 2018
Test 1 of the 2018-2019 school year

1. A large rectangle is divided into three smaller rectangles and a square as shown in the diagram. If the three nonsquare rectangles have areas 96, 120, and 180 (in some order), find all possible side lengths for the square.



Answer:_____

2. Kat writes down the units digits of the numbers $1^1, 2^2, 3^3, \ldots, 2016^{2016}, 2017^{2017}, 2018^{2018}$, obtaining a sequence $1, 4, 7, \ldots, 6, 7, 4$. What is the sum of the numbers in Kat's sequence?

Answer:_____

3. Prove that the integers 1 through 32 inclusive can be arranged around a circle so that the sum of each pair of adjacent integers is a perfect square, and that the integers 1 through n inclusive cannot be so arranged for any integer n with $2 \leq n \leq 30$.

Note: For this problem, please include your proof on separate sheets of paper.

4. If $f(n) = \dfrac{n-3}{\sqrt{2n-2}-2}$, what is the smallest integer $n > 3$ such that $f(f(f(f(n))))$ is an integer?

Answer:_____

5. Triangle ABC has $\dfrac{1}{AB} + \dfrac{1}{BC} = \cos(B) = \dfrac{1}{8}$.
Find the length of the bisector of angle B.

Answer:_____

6. Find an ordered triple (a, b, c) of positive integers with $a < b < c$ such that

$$\sqrt{\frac{1}{\sqrt{a}+\sqrt{a+1}}} + \sqrt{\frac{1}{\sqrt{b}+\sqrt{b+1}}} + \sqrt{\frac{1}{\sqrt{c}+\sqrt{c+1}}} = 1.$$

Answer:_____

The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Missisquoi Valley Union HS) and Evan Dummit (Postdoctoral Associate at Arizona State University).