Vermont State Mathematics Coalition Talent Search -- November 2018
Test 2 of the 2018-2019 school year

PRINT NAME:____ Signature:_________________
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School____ Grade _____________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **Monday, December 10, 2018** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Missisquoi Valley Union HS) and Evan Dummit (Postdoctoral Associate at Arizona State University).

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1. In a game in the National Fair-Coin-Flipping League, each flip counts for one point, and a team must score 7 points to win. If the Heads currently have 5 points and the Tails currently have 2 points, what is the probability that the Heads will win this game?

Answer:___________

2. In $\triangle ABC$, $AB = AC = 8$ and $BC = 4$. Points $D, E, \text{and } F$ are located on sides $AB,\ AC, \text{and } BC$ (respectively), such that DE is parallel to BC and $\triangle DEF$ is equilateral. Find the perimeter of $\triangle DEF$.

Answer:___________

3. Let C be the circle $x^2 + y^2 = 87$, and recall that a lattice point is a point whose coordinates are both integers. We say a lattice point is "lattice-closest" to C if no other lattice point has a smaller distance to C. The set of lattice-closest points to C forms a polygon: Determine the area of this polygon.

Answer:___________

4. If x, y, z are real or complex numbers such that $xyz = 7$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = x^3 + y^3 + z^3 = 1$, find all possible values for $x + y + z$.

Answer:___________

5. Find the sum of all values of $\theta,\ 0 \leq \theta \leq 2\pi$, for which

$$(\sin\theta + i\cos\theta)^{15} = \cos\theta + i\sin\theta$$

Answer:___________

6. Evan has a large supply of square tiles of side length $1, 2, 4, 8, 16, \ldots, 1024$. Determine, with proof, the minimum number of tiles Evan needs to cover a 2019×2019 region completely with tiles such that no tiles overlap.

Note: For this problem, please include your proof on separate sheets of paper.

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