Vermont State Mathematics Coalition Talent Search -- January 2019
Test 3 of the 2018-2019 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School_____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **Thursday, February 14, 2019** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Missisquoi Valley Union HS) and Evan Dummit (Postdoctoral Associate at Arizona State University).

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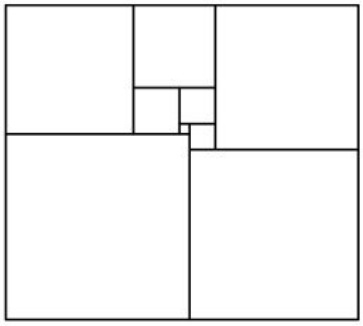
1. The diagram at right shows nine non-overlapping squares assembled to form a rectangle. The length and width of the rectangle are relatively prime positive integers. Find the perimeter of the rectangle.



Answer:_____

2. The sequence of digits $12345678910111213141516171819 20\ldots$ is obtained by writing the positive integers in order. If the 10^nth digit in this sequence occurs in the part of the sequence in which the m-digit numbers are placed, define $f(n)$ to be m. For example, $f(2) = 2$ because the 100th digit enters the sequence in the placement of the two-digit number 55. Find $f(2019)$.

Answer:_____

3. In trapezoid $ABCD$, AD is parallel to BC and $m\angle D = m\angle C - m\angle A$. If $AB = 5$, $BC = 10$, and $CD = 6$, find the area of trapezoid $ABCD$.

Answer:_____

4. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Determine the number of real numbers x with $1 < x < 2019$ and such that $\lfloor x^3 \rfloor = x^2 \lfloor x \rfloor$.

Answer:_____

5. Ken and Tony play a game involving a pile of candies. On each turn, a player may take either one-third or one-fourth of the remaining candies, rounded up to the nearest integer: thus, if there were 26 candies, a player could either remove 9 or 7 of them. Ken and Tony alternate turns, with Ken going first. In ruleset K, the player who takes the last candy wins, while in ruleset T, the player who takes the last candy loses. Starting with a pile of N candies, prove that Ken has a winning strategy in ruleset K while Tony has a winning strategy in ruleset T if and only if $N = 2^n - 1$ for some positive integer n.

Note: For this problem, please include your proof on separate sheets of paper.

6. What is the smallest positive integer d such that there exist distinct lattice points A, N, and G on the circle $x^2 + y^2 = d$ with $\cot \angle ANG = 2019$?

Answer:_____

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