Vermont State Mathematics Coalition Talent Search -- March 2019

Test 4 of the 2018-2019 school year

PRINT NAME:____ Signature:__________________

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School____ Grade _____________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **Friday, April 12, 2019** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Missisquoi Valley Union HS) and Evan Dummit (Postdoctoral Associate at Arizona State University).

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1. Find the smallest positive integer n such that for every integer m with $0 < m < 2019$ there exists an integer k for which

$$\frac{m}{2019} < \frac{k}{n} < \frac{m+1}{2020}.$$

Answer:___________

2. Each of the 8 faces of a hexagonal prism is randomly painted green, gold, black or white. Find the probability that that the prism has at least one pair of green faces that share an edge.

Answer:___________

3. In triangle ABC, $AB = 13$, $AC = 15$, and $BC = 14$. If the bisector of angle B intersects the inscribed circle of $\triangle ABC$ at points D and E, find the length of segment DE.

Answer:___________

4. If $p(x)$ is a polynomial with integer coefficients and n is an integer such that $p(n) = 3$, $p(n+3) = 24$, and $p(2n+3) = 63$, find all possible values of n.

Answer:___________

5. The value of $\cos^2(1°) + \cos^2(2°) + \cdots + \cos^2(44°)$ can be written in the form $\frac{p + \cot(q°)}{r}$, where p, q, and r are positive integers and $q < 90$. Find the ordered triple (p, q, r).

Answer:___________

6. Determine, with proof, all positive integers n with the property that there exists a way to label every lattice point in the plane with a positive integer such that

(a) Every positive integer is used to label exactly one point, and

(b) The sum of the labels of the vertices of any four points forming a square is divisible by n.

Note: For this problem, please include your proof on separate sheets of paper.

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