

Vermont State Mathematics Coalition Talent Search -- February 2020

Test 4 of the 2019-2020 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to Kiran.MacCormick@mvsdschools.org or be postmarked by **March 21, 2020** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

1. Kiran has 2020 red blocks, labeled with the integers 1 to 2020 inclusive, and Evan has 2020 blue blocks, also labeled with the integers 1 to 2020 inclusive. Kiran and Evan group their blocks into pairs, with one red block paired with one blue block, in such a way that the sum of the labels of the blocks in each pair is a power of 2. They then evaluate the product of the numbers on each pair of blocks. What is the sum of the 2020 products that Kiran and Evan obtain?

Answer: _____

2. The roots of the polynomial $3x^2 + 7x + k$ are $\sec(\theta)$ and $\tan(\theta)$ for some angle θ with $0 \leq \theta \leq 2\pi$. Determine the value of k .

Answer: _____

3. A convex pentagon has consecutive side lengths of d, d, d, d , and $5\sqrt{11}$. If there exists a circle of radius 9 passing through all of its vertices, find all possible values of d^2 .

Answer: _____

4. Let d be the greatest common divisor of $2^{2019^{2018}} - 2$ and $2^{2019^{2020}} - 2$. Compute the value of $\log_2(d + 2)$.

Answer: _____

5. Two congruent spheres with disjoint interiors are both contained inside a regular tetrahedron of side length 6. Determine the greatest possible value for the shared radius of the spheres.

Answer: _____

6. Suppose that $\sqrt{5}$ is expressed in base 3, as $1.020101_3\dots = 1.d_1d_2d_3d_4\dots$. Prove that for every positive integer n , at least one of the digits $d_n, d_{n+1}, d_{n+2}, \dots, d_{2n}$ is nonzero.

Note: For this problem, please include your proof on separate sheets of paper.