

Vermont State Mathematics Coalition Talent Search -- December 2020

Test 2 of the 2020-2021 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **January 8, 2020** and submitted to

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Champlain Valley Union High School
369 CVU Road
Hinesburg, VT 05461

To receive the next tests via email, clearly print your email address below:

1. Evan has 2020 candies and wants to give some of them to the guests who will come to his birthday party. However, Evan does not know the exact number of guests, only that there will be between 500 and 2020 guests (inclusive). Evan therefore wants to distribute the candies among the pinatas before the party in such a way that no matter how many guests will show up, Evan can select some of the pinatas for the guests to open so that each guest will get exactly one candy. One possibility is for Evan to use 2020 pinatas, place one candy in each, and then select an appropriate number of them when the guests arrive. What is the minimum number of pinatas Evan will need to buy?

Answer: _____

2. At the Armenian Raffle Meandering Luncheon (ARML), a raffle drawing is performed with tickets labeled 853158-853219 inclusive. The Vermont team coaches possess four tickets in this raffle. What is the probability that the coaches have won zero prizes after 28 tickets have been drawn? Express your answer as a percent to the nearest tenth of a percent.

Answer: _____

3. Twelve equally-spaced points are marked on a circle of radius 1. The 66 segments joining each possible pair of these points to each other are then drawn. The geometric mean of these 66 segments' lengths can be written in the form $\sqrt[n]{b}$ where a and b are positive integers and a is as small as possible. Find the sum $a + b$.

Answer: _____

4. Suppose that $p(x) = ax^3 + bx^2 + cx + d$ is a degree-3 polynomial such that the values $p(0), p(1), p(2), p(3), p(4) \dots$ are all integers. What is the minimum possible positive real number value for the coefficient a ?

Answer: _____

5. Suppose $n > 1$ is a positive integer.

(a) Prove that there does not exist an n -digit integer b such that all of the digits of b and of b^2 are odd.

(b) Prove that there does exist an n -digit integer a such that all of the digits of a and of a^2 are nonzero and even.

Note: For this problem, please include your proof on separate sheets of paper.

6. Kathleen is making circular bracelets that will have 12 evenly-spaced gems. Each gem is one of four types: amethyst, lapis lazuli, turquoise, or citrine. Bracelets are considered equivalent if they can be rotated into one another: thus, a bracelet with six citrine followed by six amethyst is equivalent to the bracelet with three citrine, six amethyst, and three citrine. Find the number of distinct bracelets Kathleen could make.

Answer: _____