

Vermont State Mathematics Coalition Talent Search -- March 2021

Test 4 of the 2020-2021 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **April 24, 2021** and submitted to

Kiran MacCormick
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369 CVU Road
Hinesburg, VT 05461

To receive the next tests via email, clearly print your email address below:

1. For each of the 336 ordered triples of three distinct vertices (A, B, C) of a cube, Evan calculates the angle measure $m\angle ABC$. In degrees, what is the sum of all the distinct angle measures on Evan's list?

Answer: _____

2. The sequence of integers a_0, a_1, a_2, \dots satisfies the conditions $a_0 = 0$ and $|a_n - a_{n-1}| = n^2$ for each positive integer n . Find the minimal possible value of k such that there exists such a sequence with $a_k = 2021$.

Answer: _____

3. Kiran is playing a version of capture-the-flag. His home base is located at the origin $(0, 0)$ and any point within a distance 1 of his home base is considered safe. Red flags are located at every point along the line $y = -\sqrt{3}$, blue flags are located at every point along the line $y = \sqrt{3}(x + 2)$, and green flags are located at every point along the line $y = -\sqrt{3}(x - 2)$. Kiran must start at a point that is safe, pick up a flag of each color, and then return to a point that is safe. What is the minimum total distance Kiran could travel while achieving this task?

Answer: _____

4. Find the smallest prime number p such that a rational number of the form a/p with $0 < a < p$ contains the string "2021" somewhere in its base-10 decimal expansion. (Please note that the 4 digits must appear in that order and cannot be separated by any other digits, but may appear anywhere in the decimal expansion.)

Answer: _____

5. Let n be an arbitrary positive integer. Prove that $1^{2021} + 2^{2021} + 3^{2021} + \dots + n^{2021}$ is divisible by $1 + 2 + 3 + \dots + n$.

Note: For this problem, please include your proof on separate sheets of paper.

6. A 4×4 grid of positive integers is called *divisibly correct* if each entry divides the entry directly above it and the entry directly to the left of it. How many divisibly correct grids are there whose upper left entry is 4042 and whose bottom right entry is 1? One such grid is shown below:

4042	4042	2021	47
4042	94	47	47
86	2	1	1
43	1	1	1

Answer: _____