

Vermont State Mathematics Coalition Talent Search -- November 2021

Test 2 of the 2021-2022 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **December 16, 2021** and submitted to

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369 CVU Road
Hinesburg, VT 05461

To receive the next tests via email, clearly print your email address below:

1. This is a relay problem. The answer to each part will be used in the next part.
- (a) A positive integer is called *anti-palindromic* if it is not a palindrome and cannot be written as the sum of two palindromes. How many two-digit anti-palindromic integers are there?
- (b) Let T be the answer to part (a). The positive integer n has exactly T positive factors $1 = d_1 < d_2 < \dots < d_T = n$. Given that $d_4 = 9$ and $d_7 = 2d_6 + 3d_2$, find the value of n .
- (c) Let S be the answer to part (b). A rectangular box is inscribed in a sphere. If the surface area of the box is S square meters and the sum of the twelve edge lengths of the box is 132 meters, what is the volume of the sphere in cubic meters?

Answers: (a) _____ (b) _____ (c) _____

2. There are 12,504,636,144,000 eighteen-digit integers that use each of the digits 1, 2, 3, ..., 9 exactly twice, and the mean of these integers is M . What is the smallest of these 12,504,636,144,000 integers that is greater than M ?

Answer: _____

3. An equiangular 12-gon has consecutive side lengths of a cm, 2012 cm, 2013 cm, 2014 cm, ..., 2021 cm, and b cm. Find the value of $a + b$.

Answer: _____

4. A positive integer N is called k -digitfriendly if N can be written as the sum of exactly k integers, not necessarily distinct, that all have the same sum of digits. For example, 31 is 2-digitfriendly because $31 = 11 + 20$, while 51 is 5-digitfriendly because $51 = 3 + 12 + 12 + 12 + 12$. Find the smallest positive integer that is both 2021-digitfriendly and 2022-digitfriendly.

Answer: _____

5. Suppose that a, b, c are real numbers such that $a + b + c = 9$ and $ab + ac + bc = 24$. Prove that $1 \leq \min(a, b, c) \leq 2$.

Note: For this problem, please include your proof on separate sheets of paper.

6. The expression $\cos(6^\circ) \cos^2(12^\circ) \tan(42^\circ) \sin^2(66^\circ)$ can be written as $A \sec^2(B^\circ)$ where A is a rational number and B is an integer with $0 \leq B \leq 89$. Compute the ordered pair (A, B) .

Answer: _____