VERMONT STATE MATHEMATICS COALITION TALENT SEARCH January 4, 2005

Test 3 of the 2004 - 2005 school year (Test 4 arrives at schools February 15, 2005)

Student Name ________________________ School ____________________________

Grade ________ Math Department Head _______________________________

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by February 1, 2005 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: http://www.state.vt.us/educ/vsmc)

1. At a farm stand there are eight bags of fruit for sale with each bag containing apples or plums, but not both. The bags contain these numbers of fruit: 17, 19, 25, 30, 34, 35, 62, and 67. Each apple costs three times as much as each plum. A customer purchased some bags of apples for \$11.88. Another customer purchased some bags of plums, also for \$11.88. After those sales, there was one bag of fruit remaining. What could the last bag be worth?

Answer: _____________________

2. Find all real numbers x which satisfy the equation: $\sqrt{\frac{5}{x^2} + \frac{2}{x} + 2} = 2 + \sqrt{\frac{5}{x^2} + \frac{2}{x} - 6}$

Answer: _____________________

3. For the complex number $n = a + bi$, where a and b are positive integers, then $n^3 + n^2 + n$ is a real number. Find the ordered pair (a, b) for which a and b are as small as possible.

Answer: _____________________

4. You are given that $m + n = 20$. The lines with equations $6y - 3x = 4$ and $|m-2|x + |n-1|y = 10$ are perpendicular. Find the coordinates of the point of intersection of the lines.

Answer: _____________________

5. If the lengths of the sides of a triangle are positive integers, then there are six non-congruent triangles that have longest side(s) 4. These six triangles have sides of length (1, 4, 4), (2, 3, 4), (2, 4, 4), (3, 3, 4), (3, 4, 4), and (4, 4, 4).

Again look at triangles whose sides whose lengths are positive integers. Looking for triangles whose longest side measures 2005: how many of them are there? Suggestion: It may help you to look at the triangles whose longest side is 1 or 3 or 5 or 7 to develop a formula.

Answer: ______________________

6. The polynomial f(x) = $x^4+(2a+1)x^3+7x^2+bx+4$ is factored into two quadratic polynomials Q(x) and P(x), each with leading coefficient 1. The roots of Q(x)= 0 are m and n, and P(m) = n and P(n) = m.

a) Find a and b.

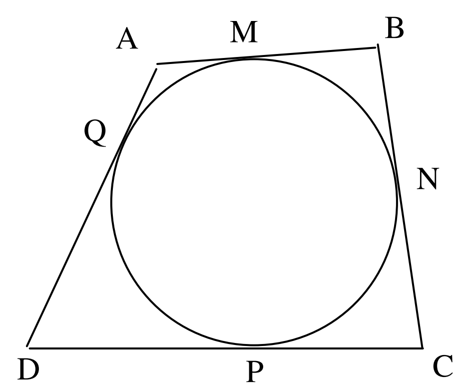
b) Find 88a + 87 b.

Answer: a) ______________ b) ________________

7. A triangle with sides of lengths 6, 10, and 14 is inscribed in a circle. Find the area of the largest triangle that can be inscribed in the circle.

Answer: ______________________

8. Refer to the diagram of the circle inscribed in a quadrilateral ABCD with the points of tangency at M, N, P, and Q. For each positive integer n, a diagram is drawn like this in which lengths of line segments are AQ = AM = n, BM = BN = n + 1, CN = CP = n + 2, and DP = DQ = n + 3.
The radius of the circle is $\sqrt{f(n)}$, where f(n) is a polynomial function in n.



Find f(n).

Hint: Let n = 1, and try to find that the radius is $r = \sqrt{5}$.

Answer: ______________________